13.6 Quadric Surfaces (part 2)

example $x^2+y^2-z^2=1$

x-ints: $x^2=1 \rightarrow x=\pm 1$

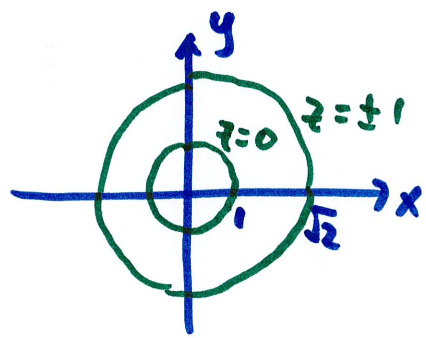
y-ints: $y=\pm 1$

z-ints: $-z^2=1 \rightarrow$ no solutions, so no z-ints

xy-trace $(z=0)$: $x^2+y^2=1$ circle radius 1

trace w/ $z=k$: $x^2+y^2=1+k^2$ circle radius $\sqrt{1+k^2}$

away from xy-plane, circles get bigger



xz-trace $(y=0)$: $x^2-z^2=1$ hyperbola vertices on x-axis

if $z=0$, $x^2=1 \rightarrow x=\pm 1$

if $x=0$, $-z^2=1 \rightarrow$ no z-ints trace w/ $y=K$ $\quad x^2-z^2=1-K^2$

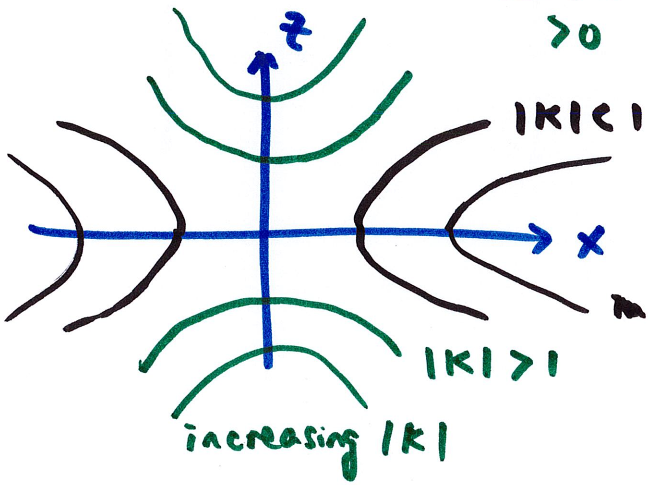
if $|K|<1$ $\quad (-1<K<1)$ $\quad 1-K^2>0$

x-ints: $\pm\sqrt{1-K^2}$

if $|K|>1$ $\quad (K<-1$ or $K>1)$ $\quad 1-K^2<0$

$$x^2-z^2=\underbrace{1-K^2}_{<0}$$

$$z^2-x^2=\underbrace{K^2-1}_{>0}$$ z-ints: $\pm\sqrt{K^2-1}$



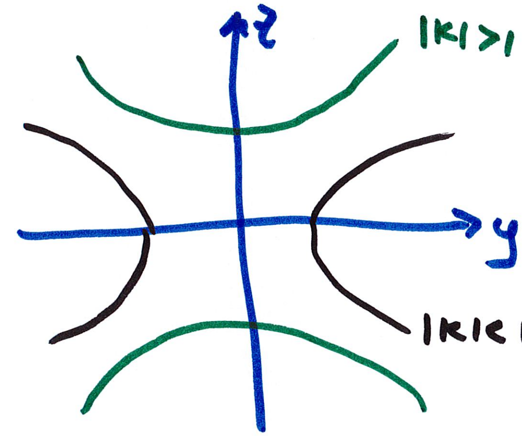
yz-slices are very similar

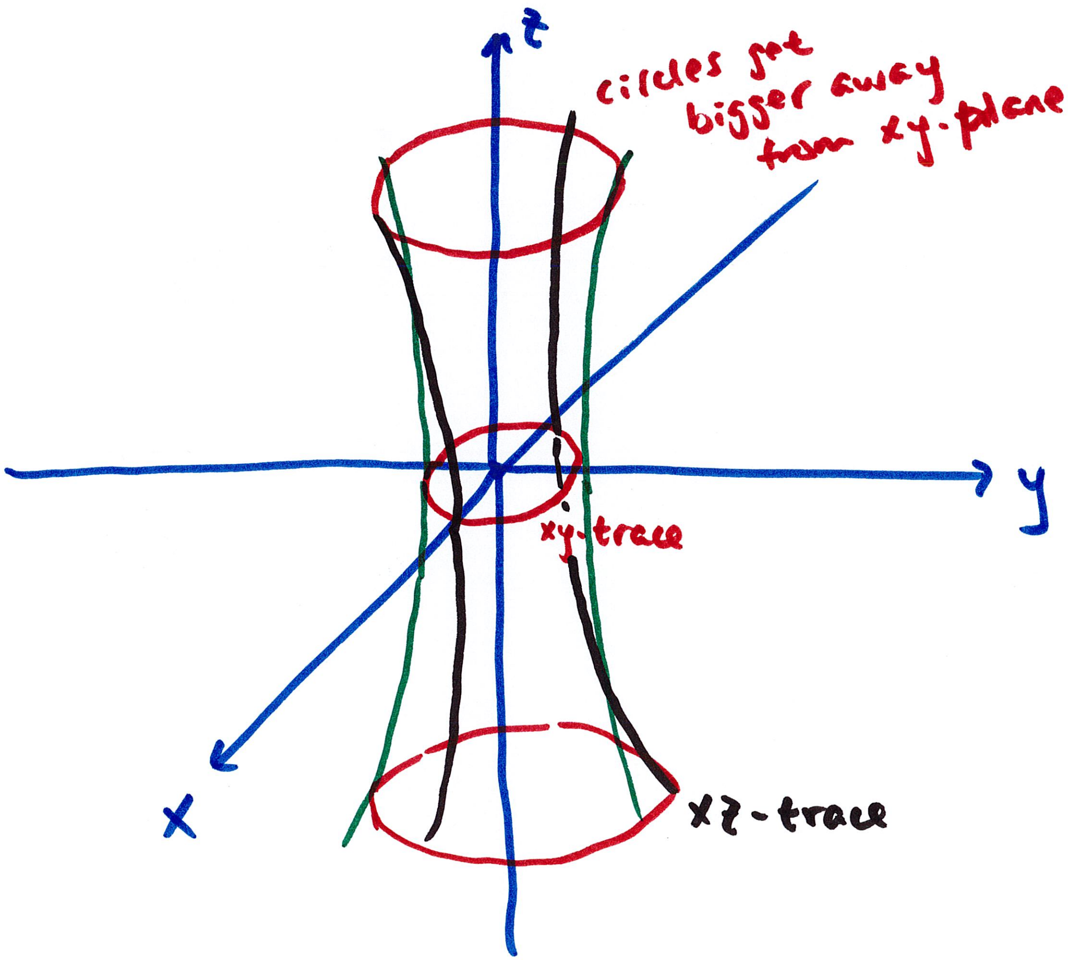
yz-trace ($x=0$) : $y^2 - z^2 = 1$ y-ints: $y = \pm 1$

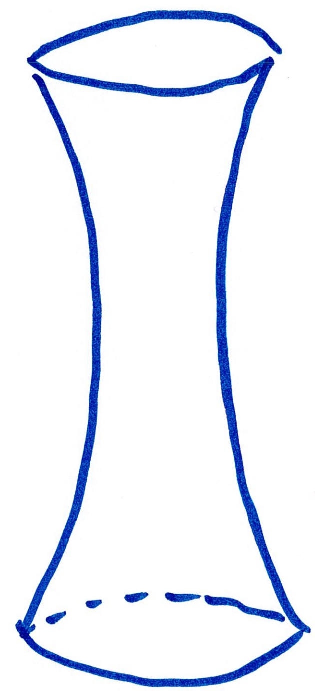
trace w/ $x=k$: $y^2 - z^2 = 1 - k^2$

if $|k|<1$ then intercept y-axis

if $|k|>1$ then intercepts z-axis

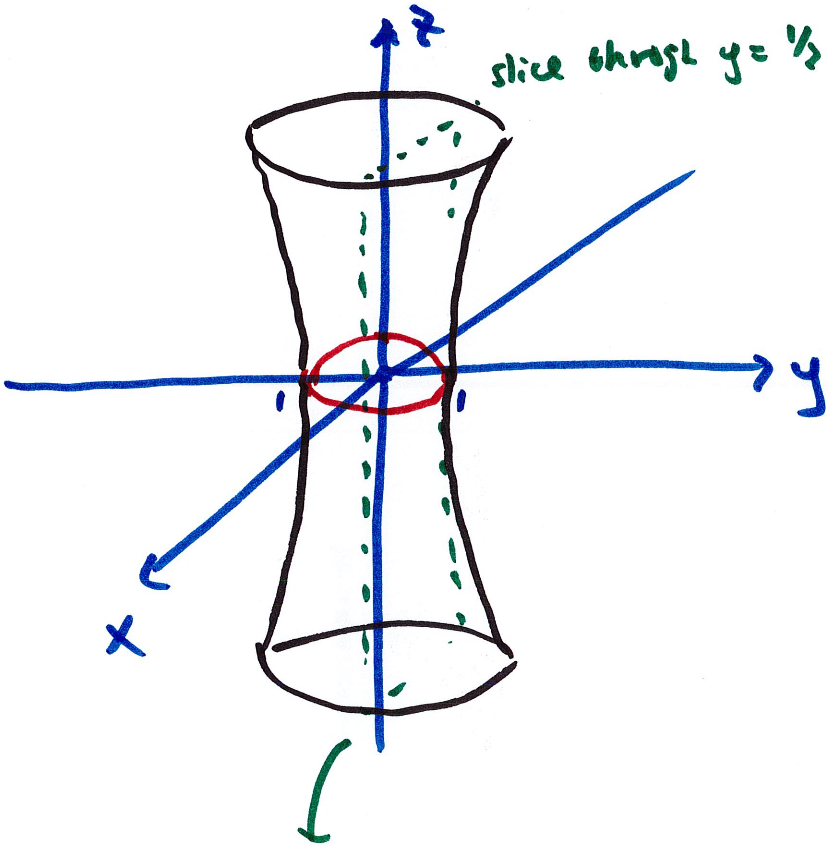


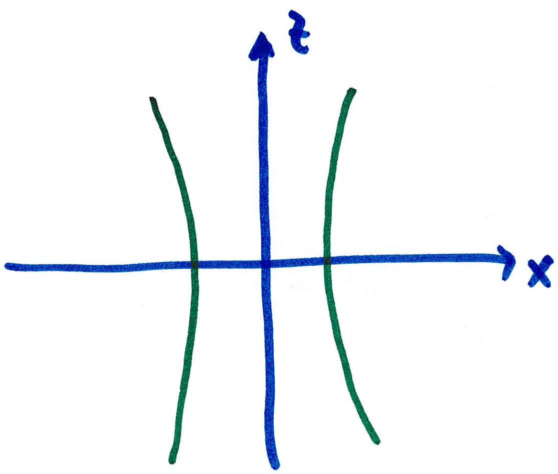




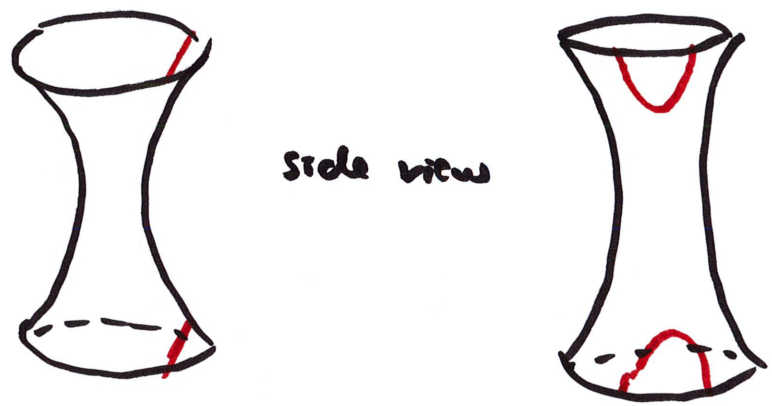
Hyperboloid of One Sheet

where are the switching hyperboles ?





slice ~~away~~ not through the "neck"
(e.g. y = 2)



example $-x^2-y^2+z^2=1$

x-ints : none $\quad -x^2=1$ has no solutions

y-ints : none

z-ints : $z=\pm 1$

xy-trace $(z=0)$: $-x^2-y^2=1 \to x^2+y^2=-1$ not a "real" shape

surface does NOT intersect xy-plane

trace w/ $z=k$: $-x^2-y^2=1-k^2$

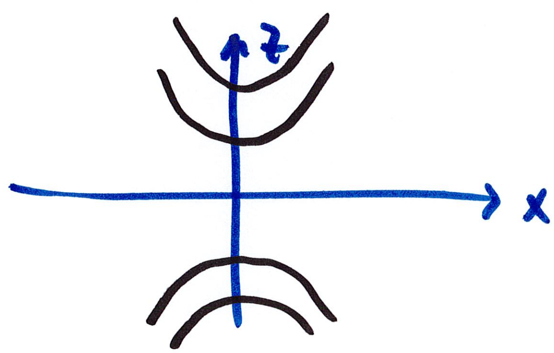
$x^2+y^2=k^2-1$ circles w/ radius $\sqrt{k^2-1}$

~~$-1\leq k\leq 1$~~ $k\geq 1,\ k\leq -1$

xz-trace $(y=0)$: $-x^2+z^2=1$ hyperbola w/ vertices at $z=\pm 1$

trace w/ $y=k$: $-x^2+z^2=1+k^2$ hyperbola w/ vertices at $z=\pm\sqrt{1+k^2}$

no switching

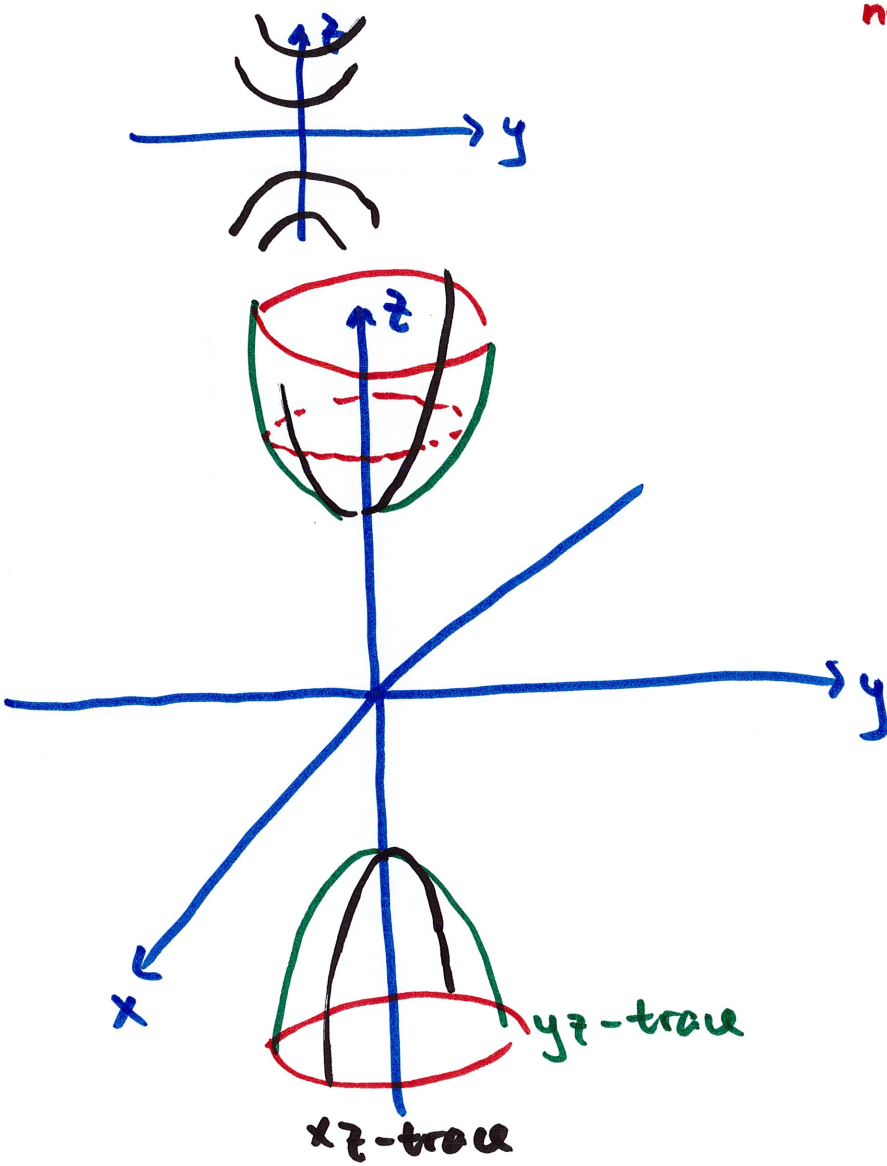


Same thing with yz-slices

yz-trace ($x=0$): $-y^2+z^2=1$ hyperbola vertices at $z=\pm 1$

trace w/ $x=k$: $-y^2+z^2=1+k^2$ " " at $z=\pm\sqrt{1+k^2}$

no switching



Hyperboloid of Two Sheets

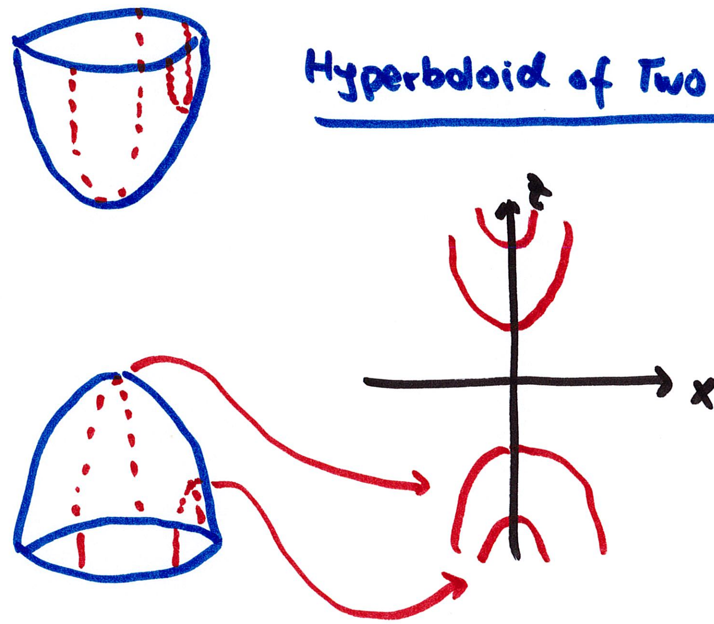
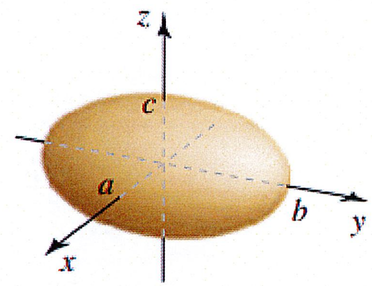
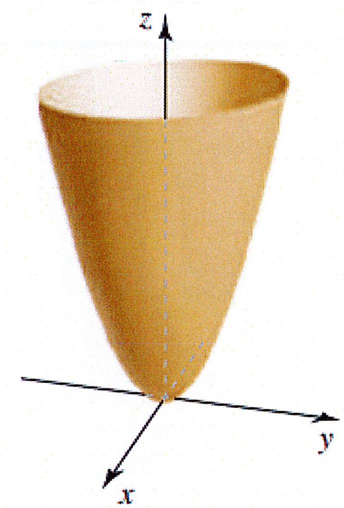
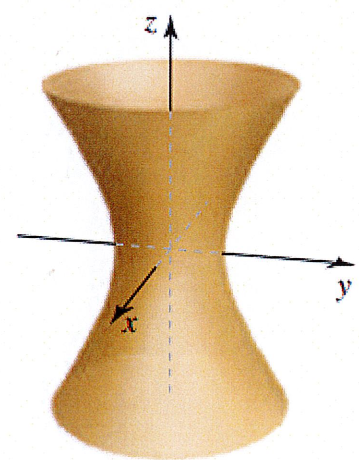


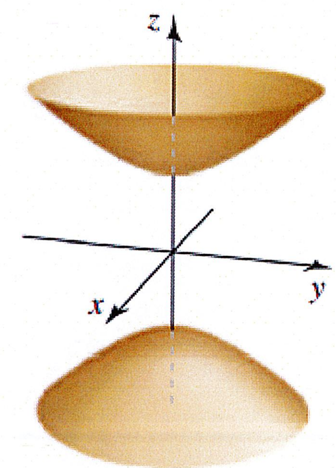
Table 13.1

Name	Standard Equation	Features	Graph
Ellipsoid	$\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} + \dfrac{z^2}{c^2} = 1$	All traces are ellipses.	
Elliptic paraboloid	$z = \dfrac{x^2}{a^2} + \dfrac{y^2}{b^2}$	Traces with $z = z_0 > 0$ are ellipses. Traces with $x = x_0$ or $y = y_0$ are parabolas.	
Hyperboloid of one sheet	$\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} - \dfrac{z^2}{c^2} = 1$	Traces with $z = z_0$ are ellipses for all z_0. Traces with $x = x_0$ or $y = y_0$ are hyperbolas.	

Hyperboloid of two sheets

$$-\frac{x^2}{a^2} - \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$$

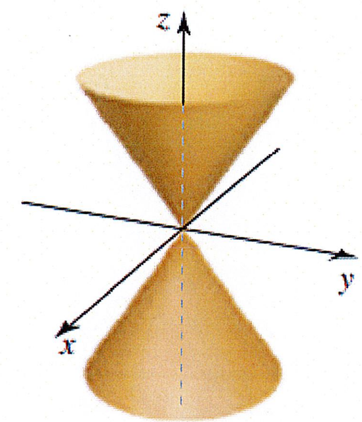
Traces with $z = z_0$ with $|z_0| > |c|$ are ellipses. Traces with $x = x_0$ and $y = y_0$ are hyperbolas.



Elliptic cone

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = \frac{z^2}{c^2}$$

Traces with $z = z_0 \neq 0$ are ellipses. Traces with $x = x_0$ or $y = y_0$ are hyperbolas or intersecting lines.



Hyperbolic paraboloid

$$z = \frac{x^2}{a^2} - \frac{y^2}{b^2}$$

Traces with $z = z_0 \neq 0$ are hyperbolas. Traces with $x = x_0$ or $y = y_0$ are parabolas.