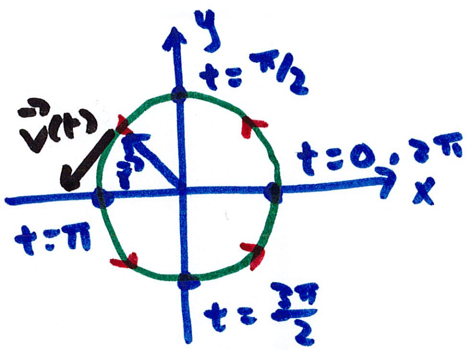
14.3 Motions in Space (part 2)

$\vec{r}(t) = \langle x(t), y(t) \rangle = \langle \cos t, \sin t \rangle \qquad 0 \le t \le 2\pi$

this is a circle because $x = \cos t$, $y = \sin t$ and $\cos^2 t + \sin^2 t = 1$

so, $x^2 + y^2 = 1$



velocity: $\vec{r}\,'(t) = \vec{v}(t) = \langle -\sin t, \cos t \rangle$

$\vec{r}(t) \cdot \vec{v}(t) = \langle \cos t, \sin t \rangle \cdot \langle -\sin t, \cos t \rangle$

$= -\cos t \sin t + \cos t \sin t = 0$

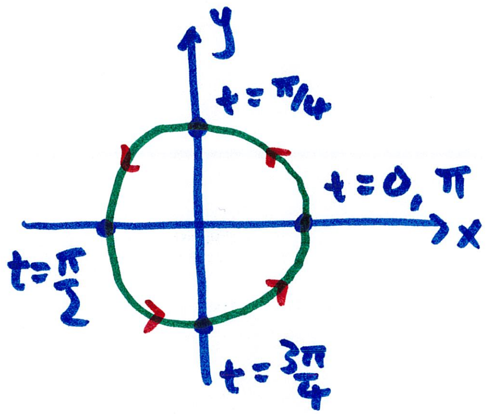
so, $\vec{r}(t) \perp \vec{v}(t)$ for all t

[illegible] $|\vec{v}(t)| = |\langle -\sin t, \cos t \rangle| = \sqrt{\sin^2 t + \cos^2 t} = \sqrt{1} = 1$

the <u>speed</u> is constant but the <u>velocity</u> is <u>NOT</u>

$\vec{r}(t) = \langle \cos(2t), \sin(2t) \rangle \qquad 0 \le t \le \pi$

notice $x^2 + y^2 = \cos^2(2t) + \sin^2(2t) = 1$ $\rightarrow$ same trajectory as before (same set of points)



Same motion but completed in half the time

$\vec{v}(t) = \langle -2\sin(2t), 2\cos(2t) \rangle$

$|\vec{v}(t)| = \sqrt{4\sin^2(2t) + 4\cos^2(2t)} = 2$

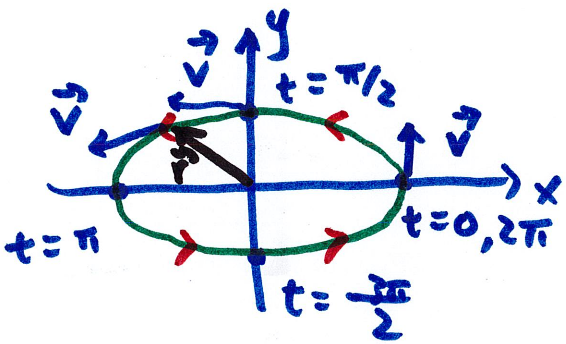
twice the speed compared to the last one

$\vec{r}(t) = \langle 2\cos t,\ \sin t \rangle \qquad 0 \le t \le 2\pi$

not circle because $x^2 + y^2 = 4\cos^2 t + \sin^2 t \ne$ constant

but $\left(\frac{x}{2}\right)^2 + (y)^2 = \cos^2 t + \sin^2 t = 1$

so, shape is $\frac{x^2}{4} + y^2 = 1$ ellipse



$\vec{v}(t) = \vec{r}\,'(t) = \langle -2\sin t,\ \cos t \rangle$

$\vec{r} \cdot \vec{v} = \langle 2\cos t, \sin t \rangle \cdot \langle -2\sin t, \cos t \rangle$

$= -4\cos t \sin t + \cos t \sin t$

$= -3\cos t \sin t \ne 0$ in general

$\vec{r}$ and $\vec{v}$ not orthogonal all the time

only orthogonal at $\cos t = 0$ or $\sin t = 0$

$\rightarrow t = 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$

$|\vec{v}| = \sqrt{4\sin^2 t + \cos^2 t} = \sqrt{3\sin^2 t + 1} \ne$ constant now we look at the motion from acceleration → path = ?

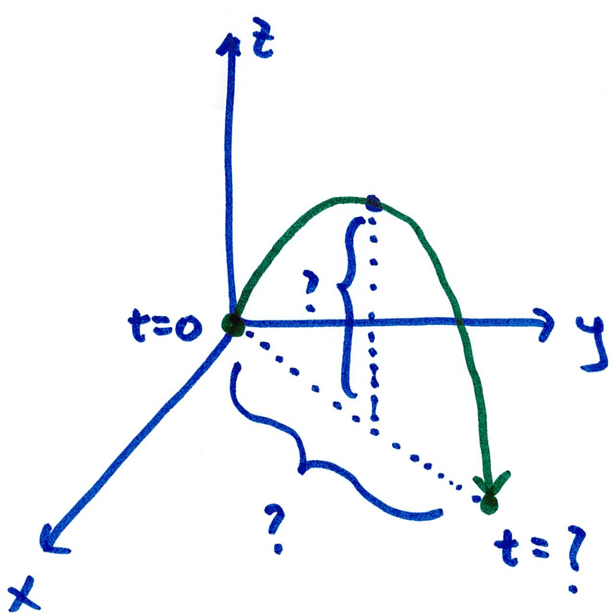
<u>example</u> A ball resting on ground is launched into the air with an initial velocity of $\vec{v}(0) = \langle 1.225, 2.45, 4.9 \rangle$ m/s.

If gravity is the only acceleration on the ball, find:

a) How long does the ball stay in the air?

b) How far does it travel?

c) How high does it go?



z is height

so, $\vec{a} = \langle 0, 0, -9.8 \rangle$ m/s²

assume $\vec{r}(0) = \langle 0, 0, 0 \rangle$

$$\vec{v}(t) = \int \vec{a}(t)\,dt = \int \langle 0, 0, -9.8 \rangle\,dt = \langle c_1, c_2, -9.8t + c_3 \rangle$$

at $t=0$, $\vec{v}(0) = \langle c_1, c_2, c_3 \rangle = \langle 1.225, 2.45, 4.9 \rangle$

so, $\boxed{\vec{v}(t) = \langle 1.225, 2.45, -9.8t + 4.9 \rangle}$

$$\vec{r}(t) = \int \vec{v}(t)\,dt = \langle 1.225t + d_1, 2.45t + d_2, -4.9t^2 + 4.9t + d_3 \rangle$$

at $t=0$, $\vec{r}(0) = \langle d_1, d_2, d_3 \rangle = \langle 0, 0, 0 \rangle$

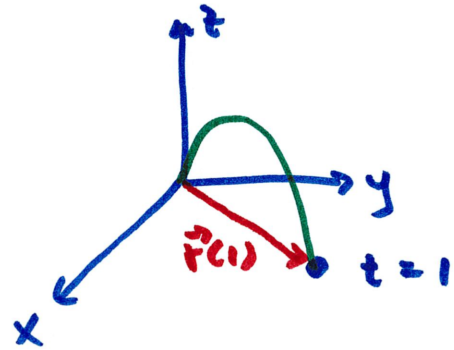
so, $\boxed{\vec{r}(t) = \langle 1.225t, 2.45t, -4.9t^2 + 4.9t \rangle}$

the time of flight : find when z-component of $\vec{r}$ is 0

$$-4.9t^2 + 4.9t = 0$$

$$4.9t(-t+1) = 0 \rightarrow t = 0,\ t = 1$$

$\boxed{\text{time of flight is 1 second}}$



range is therefore $|\vec{r}(1)|$

$\vec{r}(1) = \langle 1.225,\ 2.45,\ 0 \rangle$

$|\vec{r}(1)| = 2.74$

$\boxed{\text{range is 2.74 meters}}$

max height: when z of $\vec{v}(t)$ is 0

then use that to find z of $\vec{r}(t)$

z-velocity: $-9.8t + 4.9 = 0 \rightarrow t = \frac{1}{2}$

height: $-4.9t^2 + 4.9t = -4.9\left(\frac{1}{2}\right)^2 + 4.9\left(\frac{1}{2}\right) = 1.225$

$\boxed{\text{max height is 1.225 meters}}$

what if we ~~we~~ we doubled the initial velocity?

now $\vec{v}(0) = \langle 2.45, 4.9, 9.8 \rangle$

$\vec{a}(t) = \langle 0, 0, -9.8 \rangle$

$\vec{r}(0) = \langle 0, 0, 0 \rangle$

same steps

$$\vec{v}(t) = \int \vec{a}(t)\,dt = \cdots = \boxed{\langle 2.45, 4.9, -9.8t + 9.8 \rangle}$$

$$\vec{r}(t) = \int \vec{v}(t)\,dt = \cdots = \boxed{\langle 2.45t, 4.9t, -4.9t^2 + 9.8t \rangle}$$

time of flight: $-4.9t^2 + 9.8t = 0$

$4.9t(-t+2) = 0 \quad t=0, t=2$ time of flight is 2 seconds

(doubled)

range: $|\vec{r}(2)| = |\langle 4.9, 9.8, 0 \rangle|$

$= 10.96 \rightarrow$ 4 times of the old one