15.1 Functions of Several Variables

we are familiar with functions of one variable

for example, $y = f(x) = \underbrace{\sqrt{9-x^2}}_{\text{output}}$

input

the set of all acceptable input values is the **domain**

for this function, the domain is $9 - x^2 \geq 0$

$-3 \leq x \leq 3$ on $[-3, 3]$

for a single-variable function the domain is a line or part of lines

the set of all possible output values is the **range**

for this function, the range is $[0, 3]$

when $x = \pm 3$

when $x = 0$

now let's see a two-variable function

for example, $z = f\underbrace{(x,y)}_{\text{input}} = \underbrace{\sqrt{9-x^2} - \sqrt{25-y^2}}_{\text{output}}$

note the input is now ~~an~~ ordered pairs (x,y)

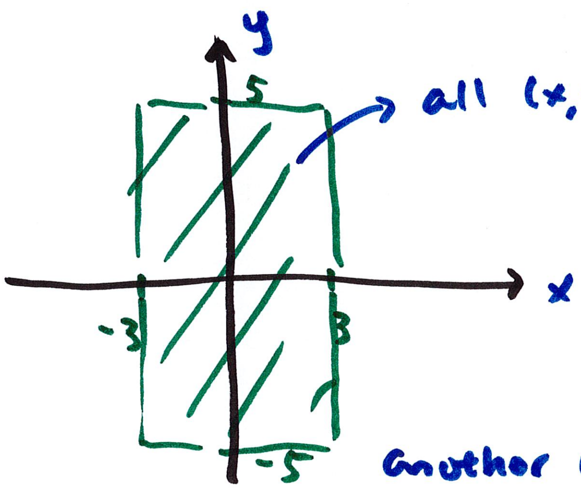
output is still a scalar

domain is still the collection of acceptable input values

here, we require $9-x^2 \geq 0$ **AND** $25-y^2 \geq 0$

$-3 \leq x \leq 3$ **AND** $-5 \leq y \leq 5$

notice the domain is now a region that contains all (x,y) such that



all (x,y) such that $-3 \leq x \leq 3$ and $-5 \leq y \leq 5$

instead of [ ] or ( ) we now have solid (including end points) or dashed lines (excluding ends)

another way to express: $\boxed{\{(x,y) : -3 \leq x \leq 3,\ -5 \leq y \leq 5\}}$

the range remains the same since the output values are scalars

$$f(x,y) = \underbrace{\sqrt{9-x^2}}_{\substack{\text{largest : } 3 \\ \text{Smallest : } 0}} - \underbrace{\sqrt{25-y^2}}_{\substack{\text{largest : } 5 \\ \text{smallest : } 0}}$$

Collectively, the largest $f(x,y)$ is 3, the smallest is -5

so, range is $-5 \le z \le 3$

or $[-5, 3]$

or $\{z : -5 \le z \le 3\}$

we already know $z = f(x,y)$ gives us a <u>surface</u> (sphere, cone, etc)

if we set $z = z_0 =$ constant, then the resulting graph $z_0 = f(x,y)$

is called a <u>level curve</u> or a <u>contour</u> of $f(x,y) = z_0$

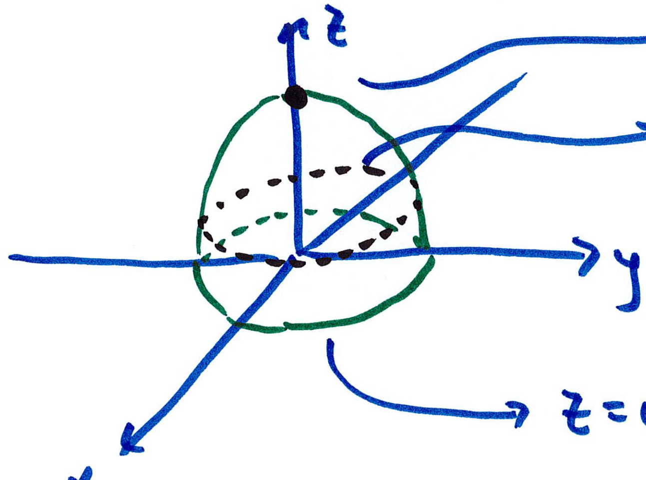
(very similar to a <u>trace</u>)

<u>example</u> $z = f(x,y) = 5 - x^2 - y^2$ paraboloid

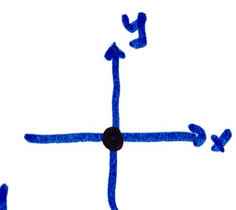
domain: $-\infty < x < \infty$ AND $-\infty < y < \infty$

or $\{(x,y) : \mathbb{R}^2\}$

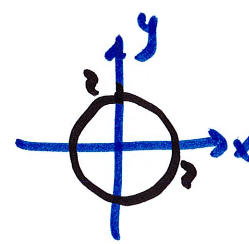
range: $-\infty < z \leq 5$



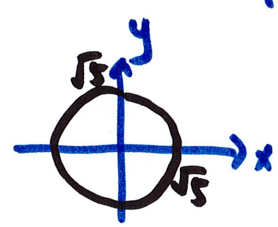
$z = 5 = 5 - x^2 - y^2 \rightarrow x^2 + y^2 = 0$



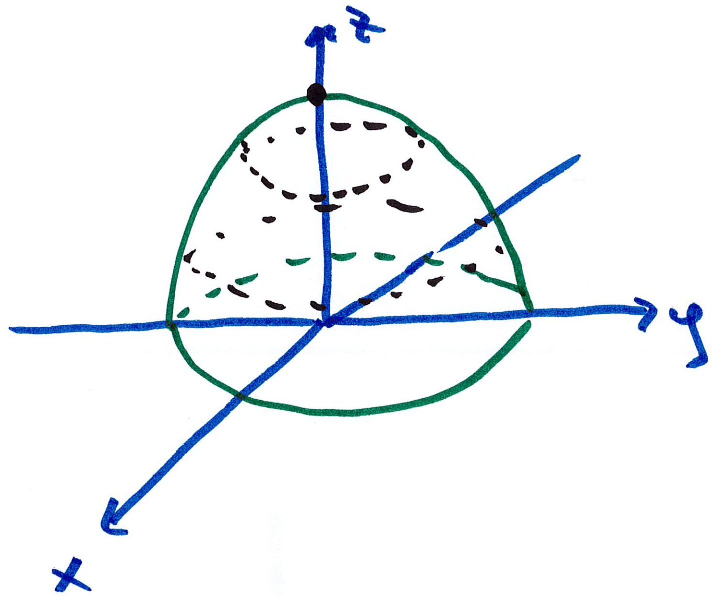
$z = 1 = 5 - x^2 - y^2 \rightarrow x^2 + y^2 = 4$

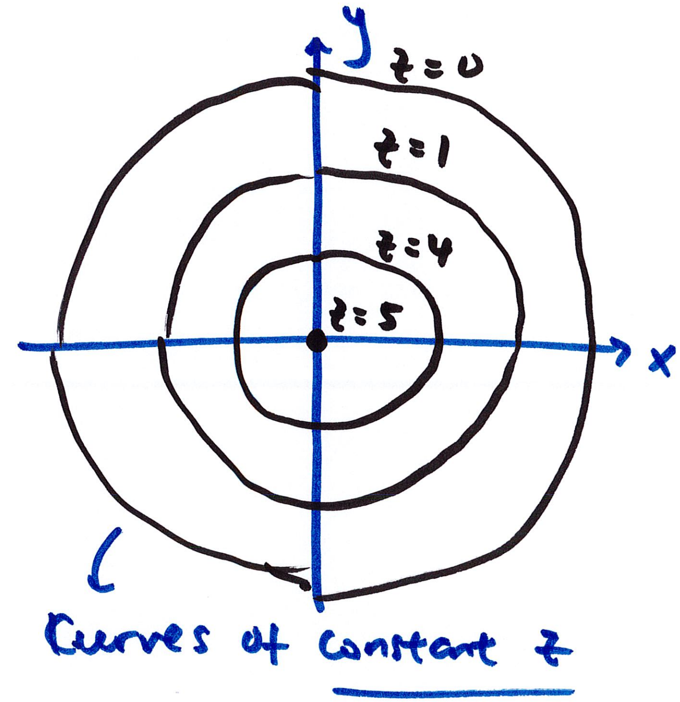


$z = 0 = 5 - x^2 - y^2 \rightarrow x^2 + y^2 = 5$



we often collect all the contours to form a contour map





example $f(x,y) = \sin(xy)$

domain: $\{(x,y) : \mathbb{R}^2\}$

range: $\{z : -1 \le z \le 1\}$

the level curves are a bit harder than the last one

let $z = z_0 = \text{constant}$

then $z_0 = \text{constant} = \sin(xy) \rightarrow xy = \underbrace{\sin^{-1}(z_0)}_{\text{constant}} \rightarrow xy = K$

$xy = K$ is a hyperbola

$\left(y = \frac{K}{x} \right.$

$y = \frac{K}{X} \rightarrow z_0 = \sin(K)$

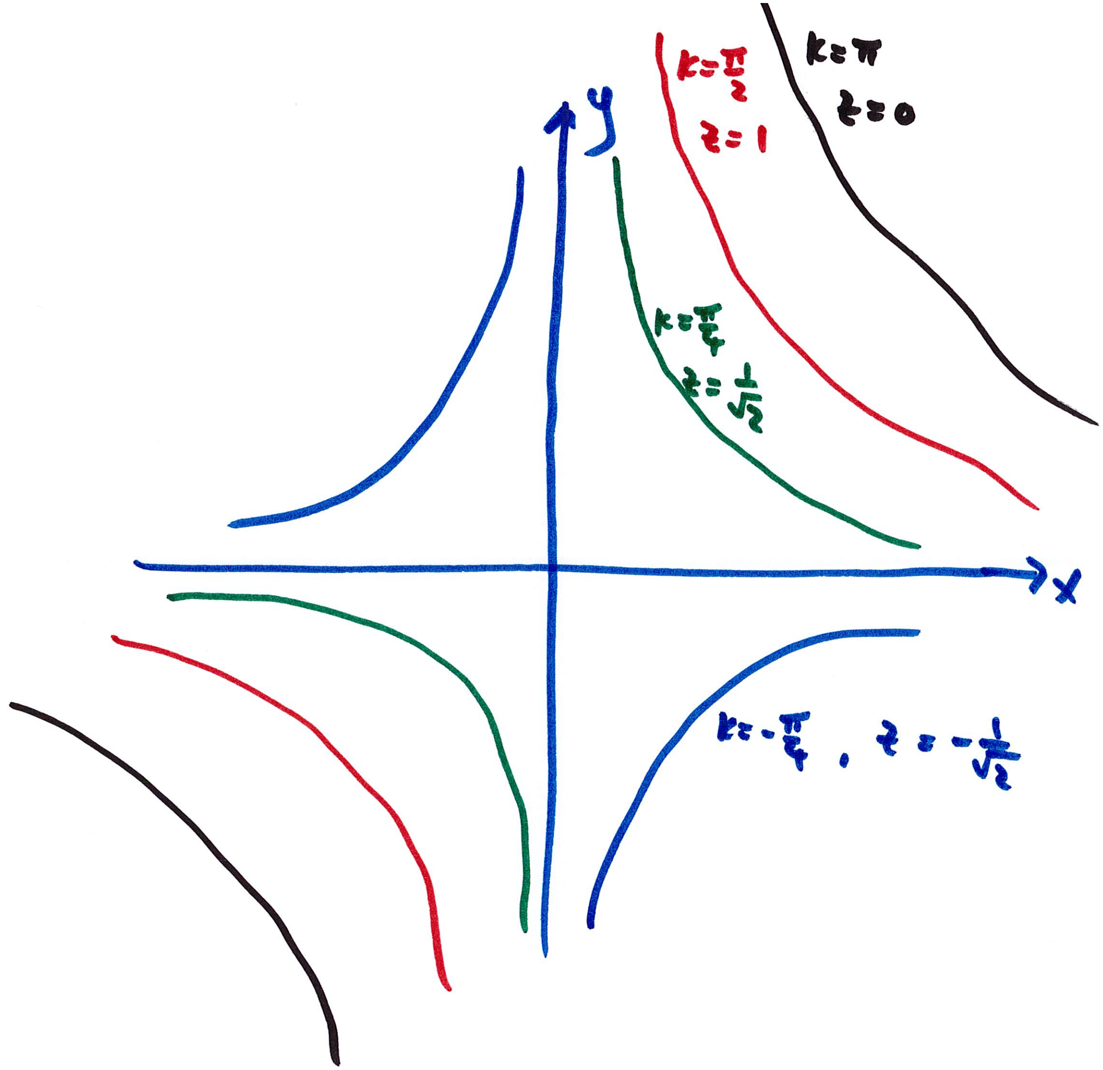
if $K = \frac{\pi}{4}$, $z_0 = \frac{1}{\sqrt{2}}$

if $K = \frac{\pi}{2}$, $z_0 = 1$

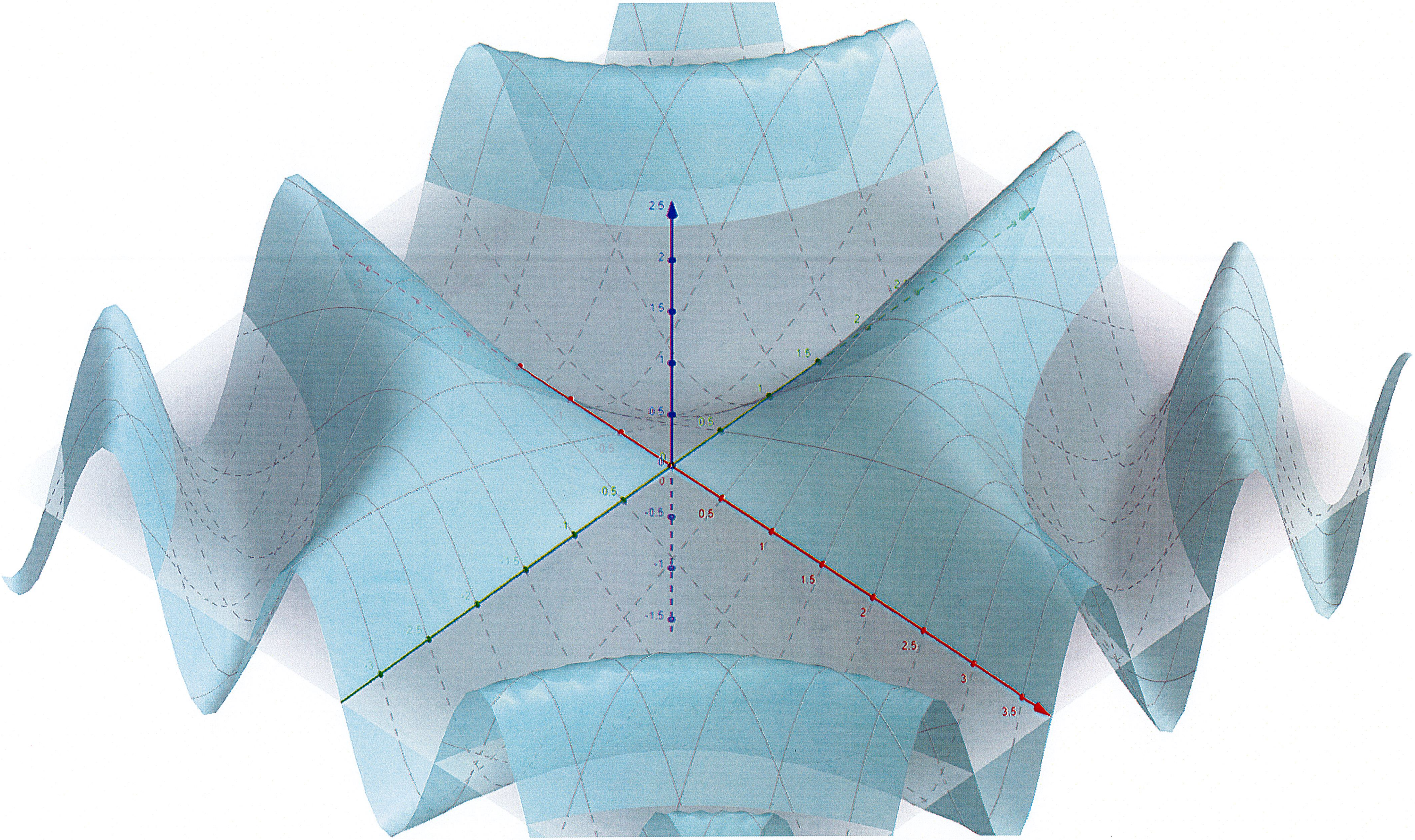
if $K = \pi$, $z_0 = 0$

if $K = -\frac{\pi}{4}$, $z_0 = -\frac{1}{\sqrt{2}}$

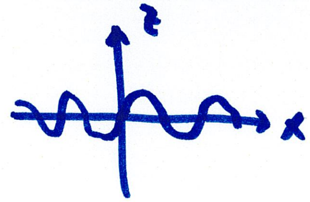
$y = \frac{-\pi/4}{X}$

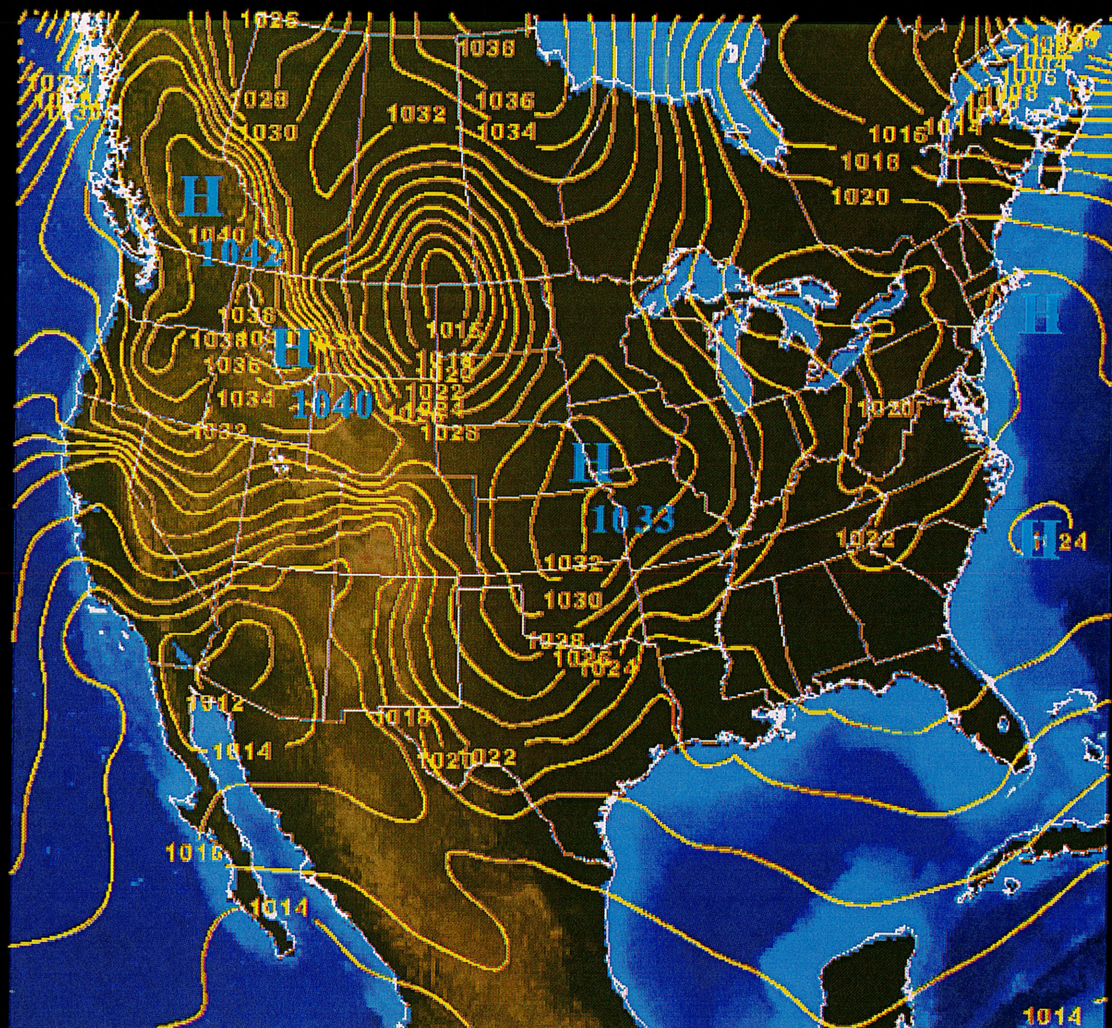


these are waves that go up and down w/ hyperbolic cross sections



side view:





SEA LEVEL PRES (mb) AND SFC WIND (m/s) ANALYSIS FOR 20021125/1400 UTC

NOAA

yellow curves are constant pressure (isobar)