# 15.3 Partial Derivatives

if $y = f(x)$ the rate of change of $y$ with respect to $x$ is

$$\frac{dy}{dx} = f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

this tells us how the changing $x$ affects $y$

if $z = f(x, y)$ now $z$ is affected by both $x$ and $y$ each of them affecting $z$

often we need to know how $x$ and $y$ individually affect $z$.

example: wind chill factor

$$W = 35.74 + 0.6215\,T - 35.75\,V^{0.16} + 0.4275\,TV^{0.16} \quad (°F)$$

T: air temp (°F)

V: wind speed (mph)

current: $T = 8$

$V = 10$

$W = -6$

$z = f(x, y)$

the partial derivative of $f$ with respect to $x$ is

weird "d"

$$\frac{\partial f}{\partial x} = f_x = \lim_{h \to 0} \frac{f(x+h, y) - f(x,y)}{h}$$

changing variable

$x$ changes while $y$ is held constant

the partial derivative of $f$ with respect to $y$ is

$$\frac{\partial f}{\partial y} = f_y = \lim_{h \to 0} \frac{f(x, y+h) - f(x,y)}{h}$$

$y$ changing
$x$ constant

In practice, we use the differentiation rules we know but pretend the non changing variable is constant.

example

$f(x,y) = x^2 + y^2 + xy$

constants

$$\frac{\partial f}{\partial x} = f_x = \frac{\partial}{\partial x}\left(x^2 + y^2 + xy\right) = 2x + 0 + y = \boxed{2x+y}$$

variable

so y is constant

constant

$$\frac{\partial f}{\partial y} = f_y = \frac{\partial}{\partial y}\left(x^2 + y^2 + xy\right) = 0 + 2y + x = \boxed{x+2y}$$

variable

x is const

<u>example</u> $z = f(x,y) = x^3 + \tan(xy)$

y is const $\frac{\partial f}{\partial x} = f_x = z_x = \frac{\partial}{\partial x}(x^3) + \frac{\partial}{\partial x}(\tan(xy))$

chain rule

$= 3x^2 + \sec^2(xy) \cdot \frac{\partial}{\partial x}(xy)$

$= \boxed{3x^2 + y\sec^2(xy)}$

x is const $\frac{\partial f}{\partial y} = f_y = z_y = \frac{\partial}{\partial y}(x^3) + \frac{\partial}{\partial y}(\tan(xy))$

$= 0 + \sec^2(xy) \cdot \frac{\partial}{\partial y}(xy)$

$= \boxed{x\sec^2(xy)}$

# Wind Chill Chart

Temperature (°F)

| Wind (mph) | Calm | 40 | 35 | 30 | 25 | 20 | 15 | 10 | 5 | 0 | -5 | -10 | -15 | -20 | -25 | -30 | -35 | -40 | -45 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 5 | 36 | 31 | 25 | 19 | 13 | 7 | 1 | -5 | -11 | -16 | -22 | -28 | -34 | -40 | -46 | -52 | -57 | -63 |
| | 10 | 34 | 27 | 21 | 15 | 9 | 3 | -4 | -10 | -16 | -22 | -28 | -35 | -41 | -47 | -53 | -59 | -66 | -72 |
| | 15 | 32 | 25 | 19 | 13 | 6 | 0 | -7 | -13 | -19 | -26 | -32 | -39 | -45 | -51 | -58 | -64 | -71 | -77 |
| | 20 | 30 | 24 | 17 | 11 | 4 | -2 | -9 | -15 | -22 | -29 | -35 | -42 | -48 | -55 | -61 | -68 | -74 | -81 |
| | 25 | 29 | 23 | 16 | 9 | 3 | -4 | -11 | -17 | -24 | -31 | -37 | -44 | -51 | -58 | -64 | -71 | -78 | -84 |
| | 30 | 28 | 22 | 15 | 8 | 1 | -5 | -12 | -19 | -26 | -33 | -39 | -46 | -53 | -60 | -67 | -73 | -80 | -87 |
| | 35 | 28 | 21 | 14 | 7 | 0 | -7 | -14 | -21 | -27 | -34 | -41 | -48 | -55 | -62 | -69 | -76 | -82 | -89 |
| | 40 | 27 | 20 | 13 | 6 | -1 | -8 | -15 | -22 | -29 | -36 | -43 | -50 | -57 | -64 | -71 | -78 | -84 | -91 |
| | 45 | 26 | 19 | 12 | 5 | -2 | -9 | -16 | -23 | -30 | -37 | -44 | -51 | -58 | -65 | -72 | -79 | -86 | -93 |
| | 50 | 26 | 19 | 12 | 4 | -3 | -10 | -17 | -24 | -31 | -38 | -45 | -52 | -60 | -67 | -74 | -81 | -88 | -95 |
| | 55 | 25 | 18 | 11 | 4 | -3 | -11 | -18 | -25 | -32 | -39 | -46 | -54 | -61 | -68 | -75 | -82 | -89 | -97 |
| | 60 | 25 | 17 | 10 | 3 | -4 | -11 | -19 | -26 | -33 | -40 | -48 | -55 | -62 | -69 | -76 | -84 | -91 | -98 |

Frostbite Times: 30 minutes, 10 minutes, 5 minutes

Wind Chill (°F) = $35.74 + 0.6215T - 35.75(V^{0.16}) + 0.4275T(V^{0.16})$

Where, T= Air Temperature (°F) V= Wind Speed (mph)

*Effective 11/01/01*

V is const $\quad \dfrac{\partial w}{\partial T} = 0.6215 + 0.4275\, V^{0.16}$

rate of change of wind chill as temp changes w/ V held constant

follow a particular row

$\dfrac{\partial w}{\partial V} = \ldots =$ rate of change of w as wind speed varies

follow a particular column

__example__ $f(x,y) = e^x \sin(y)$

$$\left.\begin{aligned} \frac{\partial f}{\partial x} &= f_x = e^x \sin(y) \\ \frac{\partial f}{\partial y} &= f_y = e^x \cos(y) \end{aligned}\right\} \text{first-order partial derivs.}$$

second-order partial derivs

$$\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial^2 f}{\partial x^2} = f_{xx} = \frac{\partial}{\partial x}\left(e^x \sin y\right) = e^x \sin(y)$$

$$\frac{\partial}{\partial y}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial^2 f}{\partial y^2} = f_{yy} = \frac{\partial}{\partial y}\left(e^x \cos y\right) = -e^x \sin(y)$$

$$\frac{\partial}{\partial y}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial^2 f}{\underbrace{\partial y \partial x}_{\text{note order is "backwards"}}} = \underbrace{f_{xy}}_{\text{order is right}} = \frac{\partial}{\partial y}\left(e^x \sin y\right) = e^x \cos(y)$$

$$\frac{\partial}{\partial x}\left(\frac{\partial f}{\partial y}\right) = \frac{\partial^2 f}{\partial x \partial y} = f_{yx} = \frac{\partial}{\partial x}\left(e^x \cos y\right) = e^x \cos(y)$$

"mixed partials"

mixed partials are equal if the four partial derivatives are continuous at the point

<u>example</u> $f(x,y) = e^{x^2y}$

$$\frac{\partial f}{\partial x} = f_x = 2xy\,e^{x^2y}$$

$$\frac{\partial f}{\partial y} = f_y = x^2e^{x^2y}$$

$$\frac{\partial^2 f}{\partial x^2} = f_{xx} = \frac{\partial}{\partial x}\left(\frac{\partial f}{\partial x}\right) = \frac{\partial}{\partial x}\left((2xy)(e^{x^2y})\right) = e^{x^2y}\cdot 4x^2y^2 + e^{x^2y}\cdot 2y$$

product of functions of x and y → product rule

$$= (2xy)\cdot\frac{\partial}{\partial x}(e^{x^2y}) + (e^{x^2y})\cdot\frac{\partial}{\partial x}(2xy)$$

$$= (2xy)\cdot e^{x^2y}\cdot 2xy + e^{x^2y}\cdot 2y = 4x^2y^2e^{x^2y} + 2ye^{x^2y}$$

$$f_{xy} = \frac{\partial}{\partial y}(f_x) = \frac{\partial}{\partial y}\left((2xy)(e^{x^2y})\right)$$

$$= (2xy)\cdot\frac{\partial}{\partial y}(e^{x^2y}) + (e^{x^2y})\cdot\frac{\partial}{\partial y}(2xy)$$

$$= (2xy)\cdot e^{x^2y}\cdot(x^2) + (e^{x^2y})\cdot(2x)$$

$$= 2x^3ye^{x^2y} + 2xe^{x^2y} = f_{yx}$$

<u>example</u> $f(x,y,z) = xyz$

find $f_{xyz}$ and $f_{yxz}$

$f_x = yz$

$f_y = xz$

$f_z = xy$

$f_{xy} = z$ $\quad$ $f_{yx} = z$

$\boxed{f_{xyz} = 1 \qquad f_{yxz} = 1}$ mixed partials are the same

$f_{zzz} = ?$

$f_{zz} = 0 = \frac{\partial}{\partial z}(xy)$

$\frac{\partial}{\partial z}(0) = 0$

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