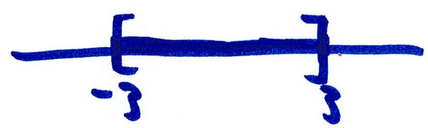
15.1 Functions of Several Variables

if $\quad y = f(\underset{\text{input}}{x}) = \underbrace{\sqrt{9-x^2}}_{\text{output}}$

the set of all acceptable input values ~~are~~ form the domain for this function. the domain is $\quad 9 - x^2 \geq 0$

$$-3 \leq x \leq 3 \quad \text{or} \quad [-3, 3]$$



note this is a line or portions of a line

the set of all possible output values is the range

here, the range is $[0, 3]$

now let's look at a function of two variables

$$z = f\underbrace{(x,y)}_{\text{input}} = \underbrace{\sqrt{9-x^2} - \sqrt{25-y^2}}_{\text{output}}$$

the input is now a set of ordered pairs (x, y)

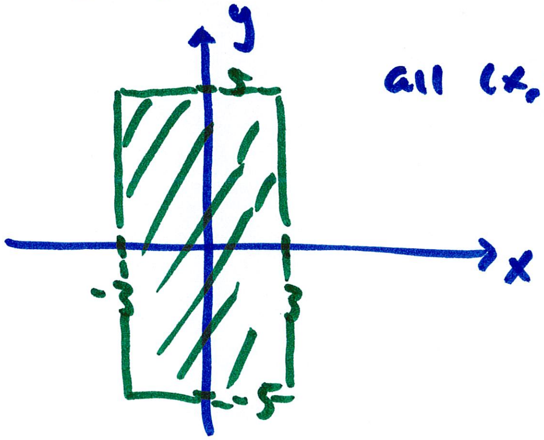
the output remains the same as before

the domain is now more complicated :

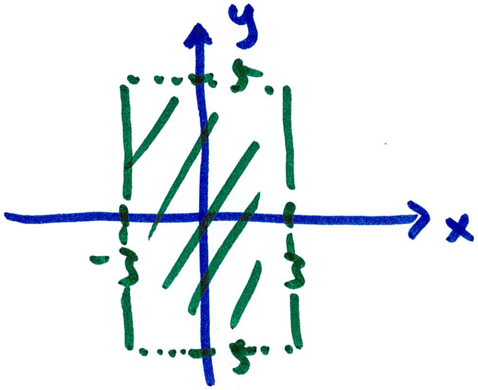
here, we need $9-x^2 \geq 0$ AND $25-y^2 \geq 0$

$-3 \leq x \leq 3$ AND $-5 \leq y \leq 5$

the graph of the domain is now a <u>region</u>



all (x,y) inside and on the boundary of this box if boundary values are not included, for example $-3 \le x \le 3$, $-5 < y < 5$



the range doesn't change from 1-variable function

$$z = f(x,y) = \underbrace{\sqrt{9-x^2}} - \underbrace{\sqrt{25-y^2}}$$

largest 3 largest 5

smallest 0 smallest 0

collectively, largest z is 3

smallest z is -5

range : $-5 \le z \le 3$

or $[-5, 3]$

we know $z = f(x, y)$ produces a surface (e.g. plane, paraboloid, etc)

to aid visualization, we sometimes use <u>level curves</u> or <u>contours</u>

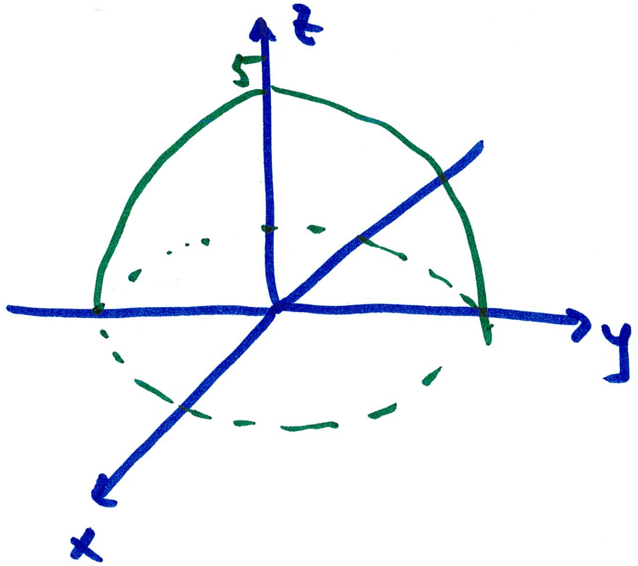
these are produced from $z = z_0 =$ constant then graph $f(x,y) = z_0$

very similar to a trace

<u>example</u> $z = f(x,y) = 5 - x^2 - y^2 = 5 - (x^2 + y^2)$ paraboloid

$z = 5 - (x^2 + y^2)$

if $z = z_0$, then we get $x^2 + y^2 = 5 - z_0$ circles radius $\sqrt{5 - z_0}$

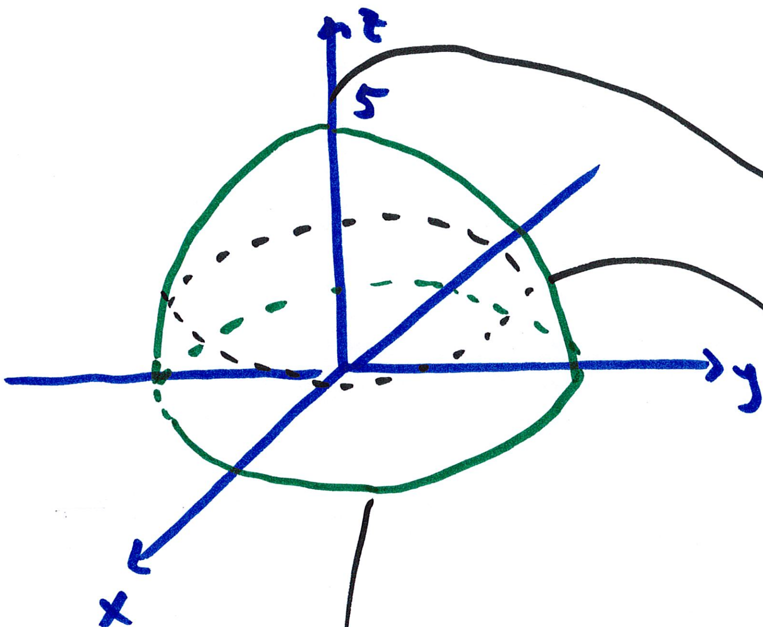


domain : $-\infty < x < \infty$
$-\infty < y < \infty$ $\rightarrow \{(x,y) : \mathbb{R}^2\}$

range : $(-\infty, 5]$

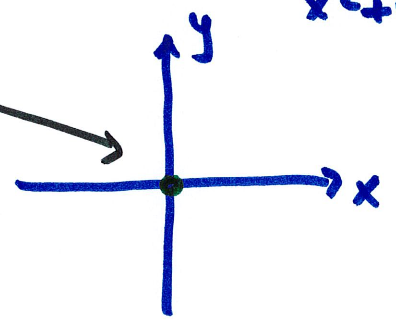
$z = f(x,y) = 5 - x^2 - y^2$

level curve / contour : set $z = z_0 = \text{constant}$



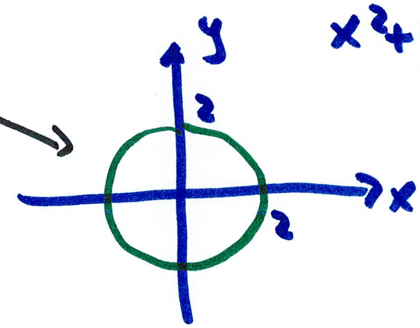
$z = z_0 = 5 \rightarrow 5 = 5 - x^2 - y^2$

$x^2 + y^2 = 0$ point

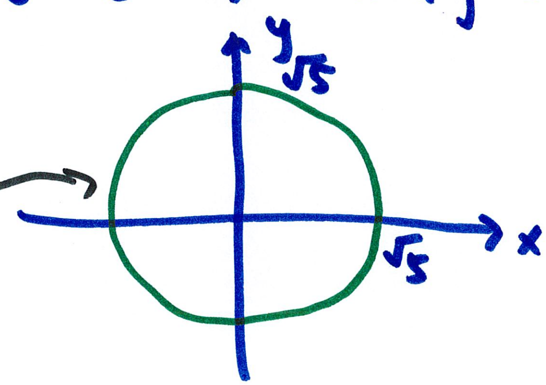


$z = z_0 = 1 \rightarrow 1 = 5 - x^2 - y^2$

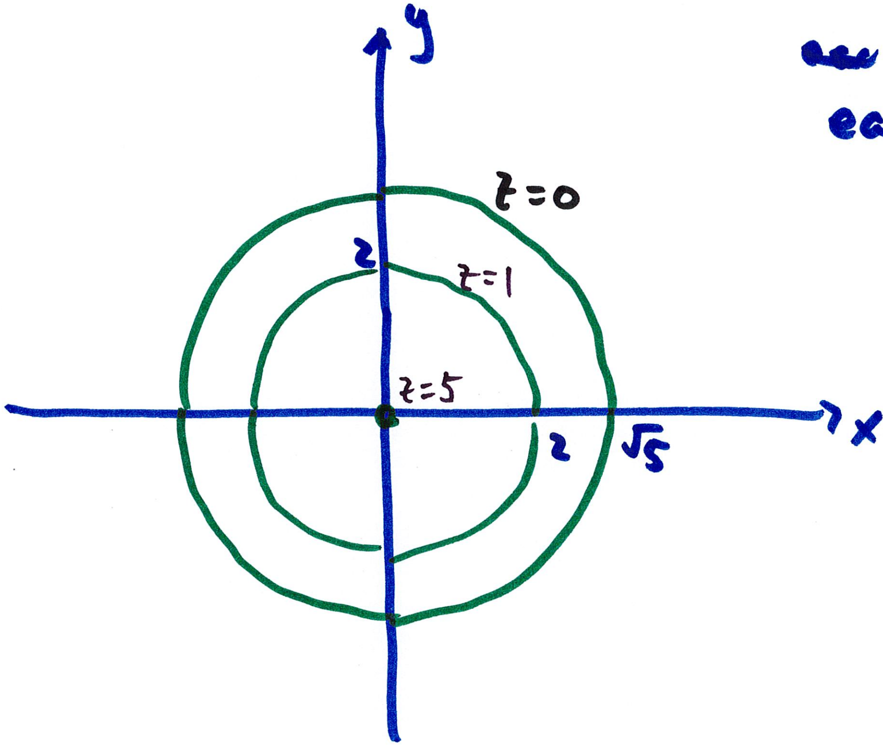
$x^2 + y^2 = 4$ circle radius 2



$z = z_0 = 0 \rightarrow x^2 + y^2 = 5$



the collection of level curves provide another way to visulize/understand the function



each curve represents the surface at a particular height

example $f(x,y) = \sin(xy)$

domain: $\{(x,y) : \mathbb{R}^2\}$ because sine can take any real number

range: $[-1, 1]$

level curves: $z = z_0$

$z = f(x,y) = \sin(xy)$

$$z_0 = \sin(xy)$$

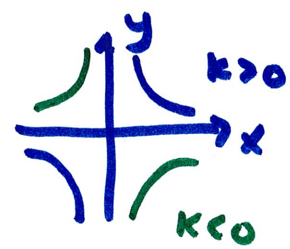
$$xy = \overbrace{\sin^{-1}(z_0)}^{\text{constant}} = k$$

constant

so level curves are $xy = k$

$$y = \frac{k}{x}$$

hyperbolas



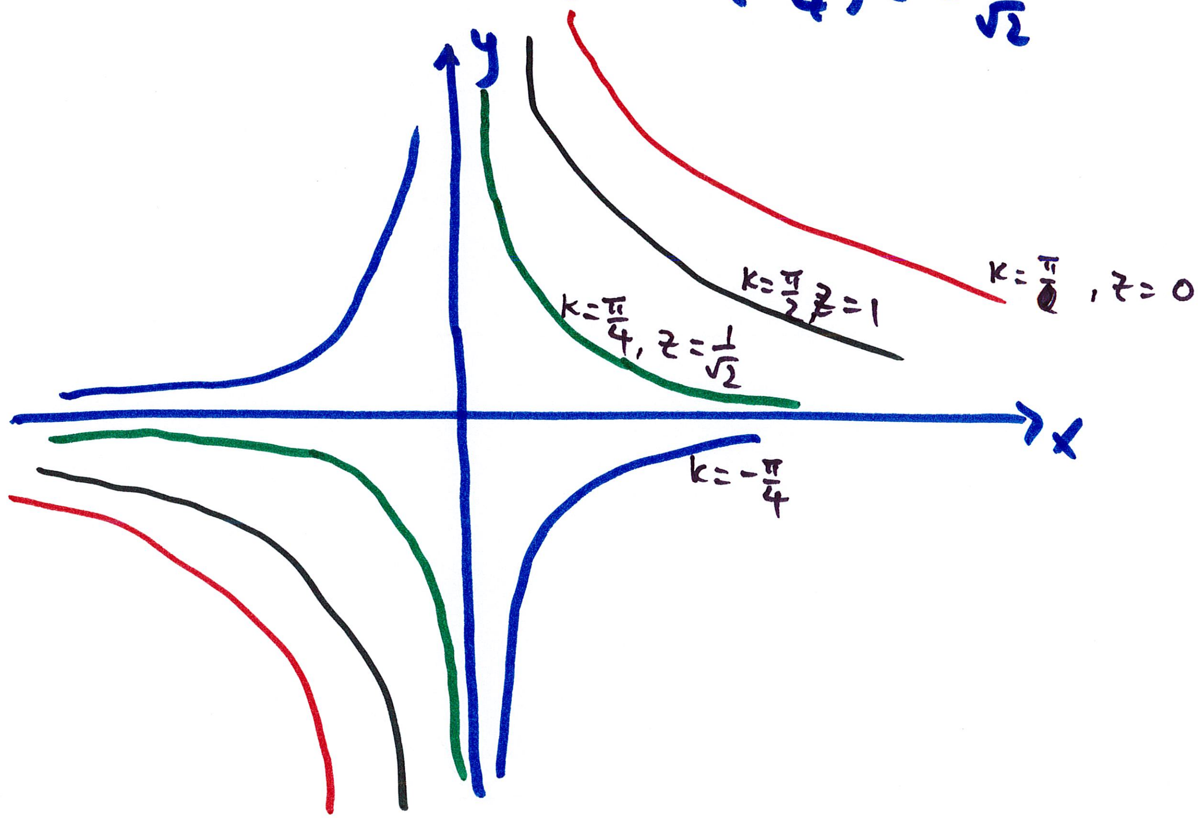
$y = \frac{k}{x}$ where $z_0 = \sin(k) \longleftrightarrow k = \sin^{-1}(z_0)$

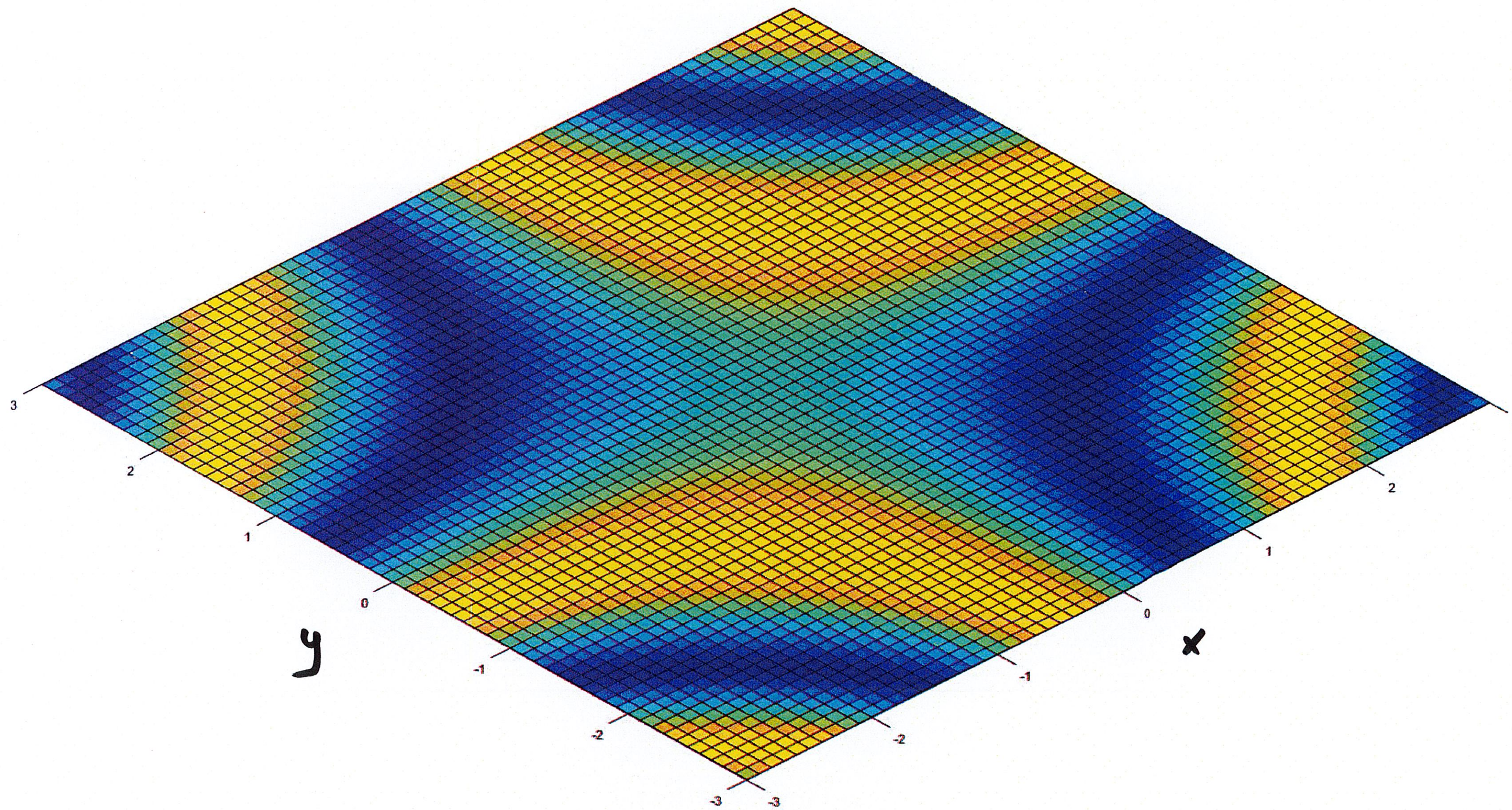
if $k = \frac{\pi}{4}$ this corresponds to $z_0 = \sin\left(\frac{\pi}{4}\right) = \frac{1}{\sqrt{2}}$

if $k = \frac{\pi}{2}$ " " " $z_0 = \sin\left(\frac{\pi}{2}\right) = 1$

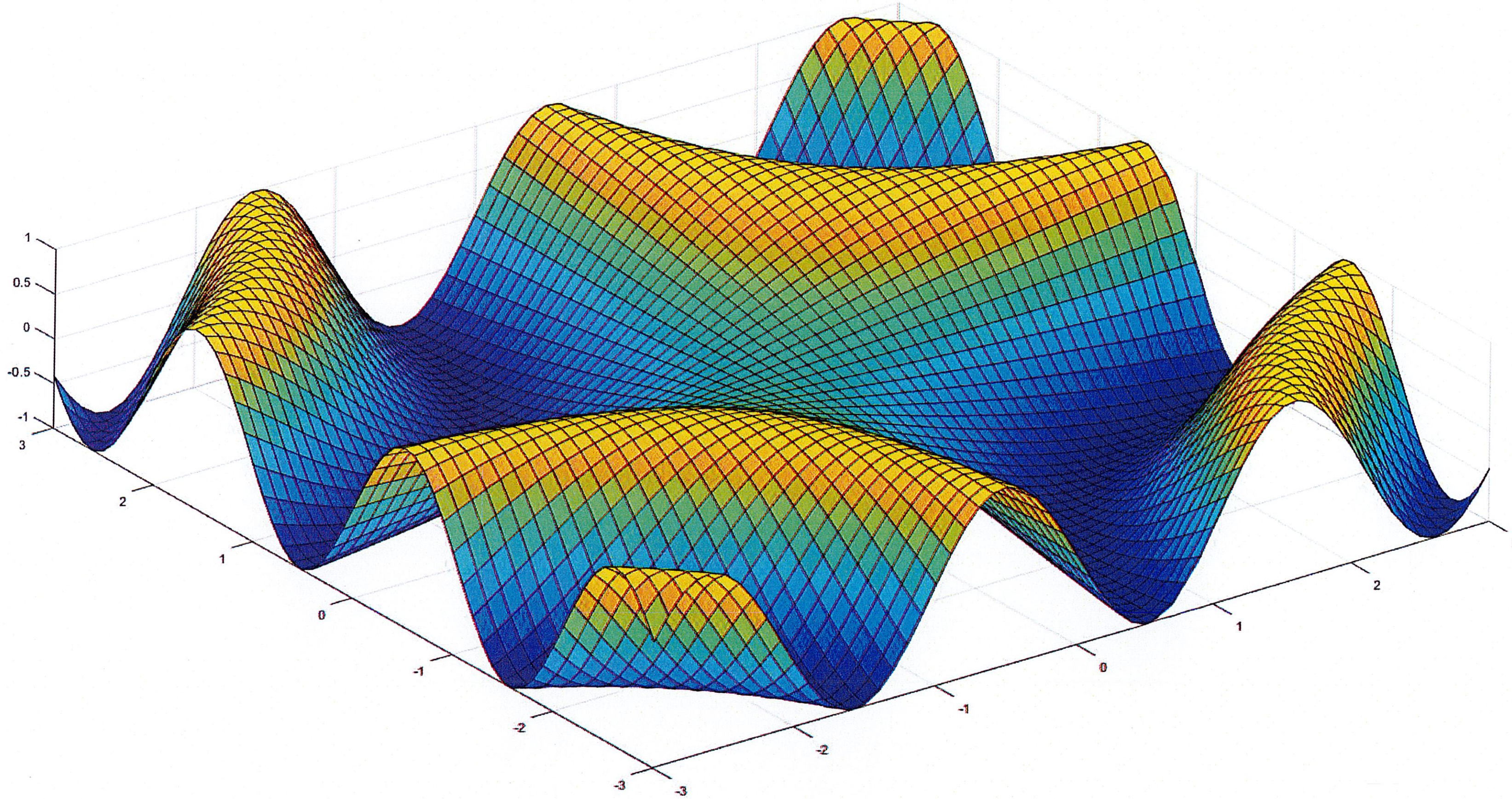
if $k = \pi$ " " " $z_0 = \sin(\pi) = 0$

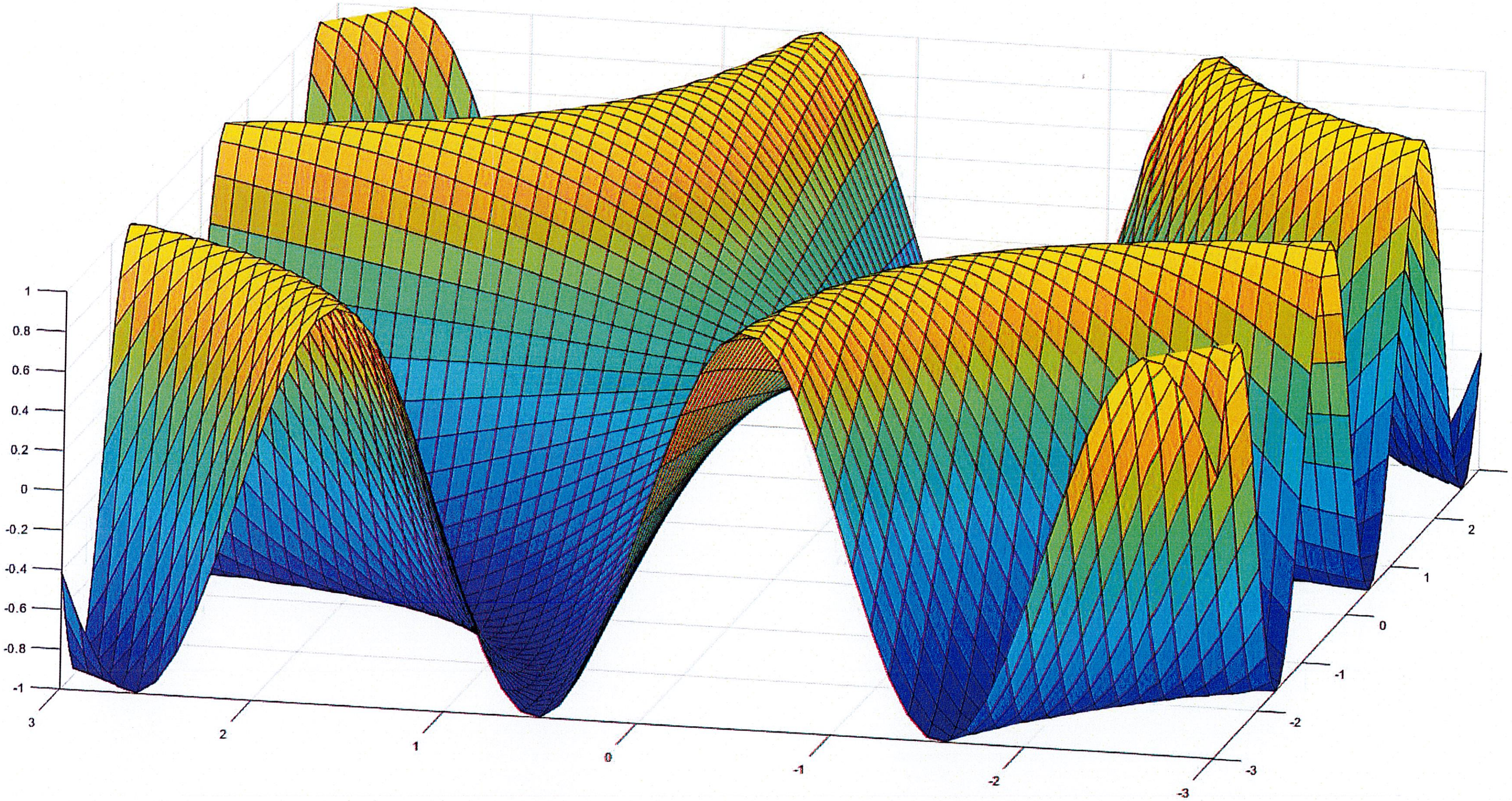
if $k = -\frac{\pi}{4}$ " " " $z_0 = \sin\left(-\frac{\pi}{4}\right) = -\frac{1}{\sqrt{2}}$





different colors are different heights


1
0.5
0
-0.5
-1
3
2
1
0
-1
-2
-3
-3
-2
-1
0
1
2
3



"edge" on with x or y pointing at us → x = constant = C

$z = \sin(Cy)$