1.1 Intro to Differential Equations

differential equation: an equation that contains a derivative

for example, $\frac{dy}{dx} = f(x)$ or $y' = f(x)$

e.g. $\frac{dy}{dx} = \cos x \rightarrow y = \sin x + C$ (calculus!)

another example $\frac{d^2r}{dt^2} = -\frac{GM}{r^2}$ Newton's Law of Gravitation

goal: find $r(t)$

one more: $\frac{dP}{dt} = K(L-P)$ Logistic Growth

$P(t)$ is population

L : pop. capacity

goal: find solution

order of differential eq: highest derivative

$y' = y$ 1st-order

$r'' = -\frac{GM}{r^2}$ 2nd-order

solution: a function that satisfies the diff. eq.

for example, $\underbrace{y' = y}_{}$ solution: $y = ?$

y is its own derivative $\to e^x$

in fact, any multiple

so, $y = Ce^x$

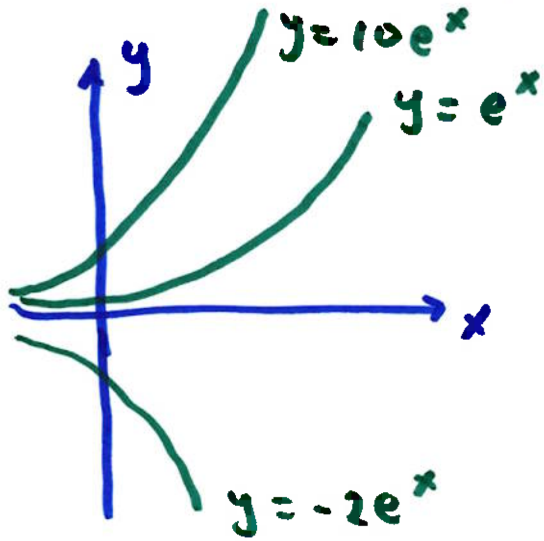
we can verify that: it must satisfy $y' = y$

does $y = Ce^x$ satisfy $y' = y$?

$$\left.\begin{array}{ll} y' = Ce^x & \text{(Left)} \\ y = Ce^x & \text{(right)} \end{array}\right\}$$

yes.

each C defines a solution curve



<u>example</u> : Is $y = \ln(x + c)$ is solution to $e^y y' = 1$?

$$e^y y' = 1$$

$$y = \ln(x+c)$$

$$y' = \frac{1}{x+c} \qquad e^y = e^{\ln(x+c)} = x+c$$

LHS: $e^y y' = x+c \cdot \frac{1}{x+c} = 1$

RHS: 1

So, $y = \ln(x+c)$ is a solution

follow-up question : find C such that the solution goes thru $(0,0)$

$$y = \ln(x+c) \qquad (\underset{x}{0}, \underset{y}{0})$$

$$0 = \ln(0+c) = \ln(c) \quad \text{so} \quad C = 1$$

each C defines a new curve based on $\underbrace{y(0) = \overset{\text{something}}{\cancel{\text{somethin}}}}_{\text{initial condition}}$

example $y'' + y' - 2y = 0$

find all values of r such that $y = e^{rx}$ is a solution.

again, if $y = e^{rx}$ is a solution, then $y'' + y' - 2y = 0$

$$\left.\begin{aligned} y &= e^{rx} \\ y' &= re^{rx} \\ y'' &= r^2 e^{rx} \end{aligned}\right\} \rightarrow y'' + y' - 2y = 0$$

$$r^2 e^{rx} + re^{rx} - 2e^{rx} = 0 \qquad r = ?$$

$$(e^{rx})(r^2 + r - 2) = 0$$

$$\cancel{e^{rx} = 0} \quad \text{or} \quad r^2 + r - 2 = 0 \rightarrow (r+2)(r-1) = 0$$

~~e^{anth}~~

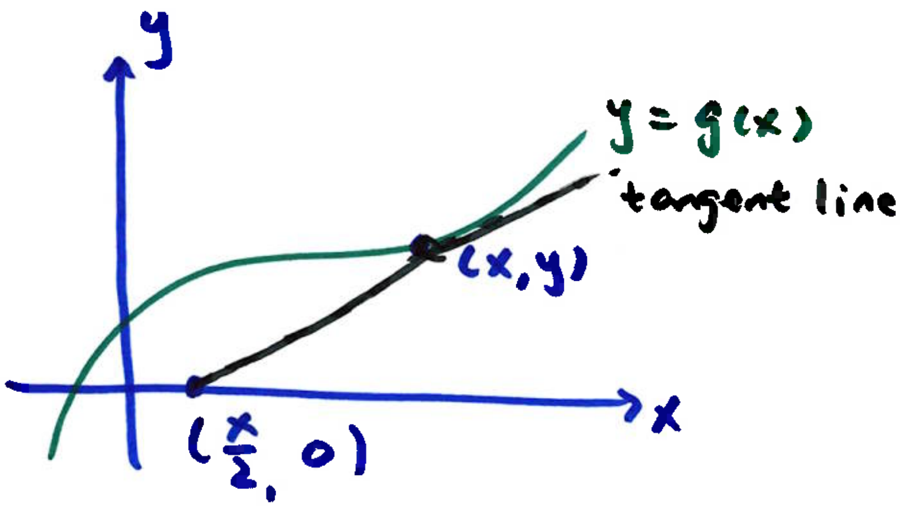
$$e^{anything} \neq 0 \qquad \boxed{r = -2 \text{ or } r = 1}$$

so, e^{x} and e^{-2x} are solutions

<u>example</u> : Write a differential eq. $\frac{dy}{dx} = f(x,y)$

such that the line tangent to the solution $y = g(x)$

at (x,y) goes thru $(\frac{x}{2}, 0)$



$\frac{dy}{dx} = ?$ that describes what we see on the left

$\frac{dy}{dx}$ is the slope of line tangent to y

tangent line goes through (x,y) and $(\frac{x}{2}, 0)$

so slope is $\frac{y-0}{x-\frac{x}{2}} = \frac{y}{\frac{x}{2}} = \frac{2y}{x}$

so the diff. eq is

$$\boxed{\frac{dy}{dx} = \frac{2y}{x}}$$