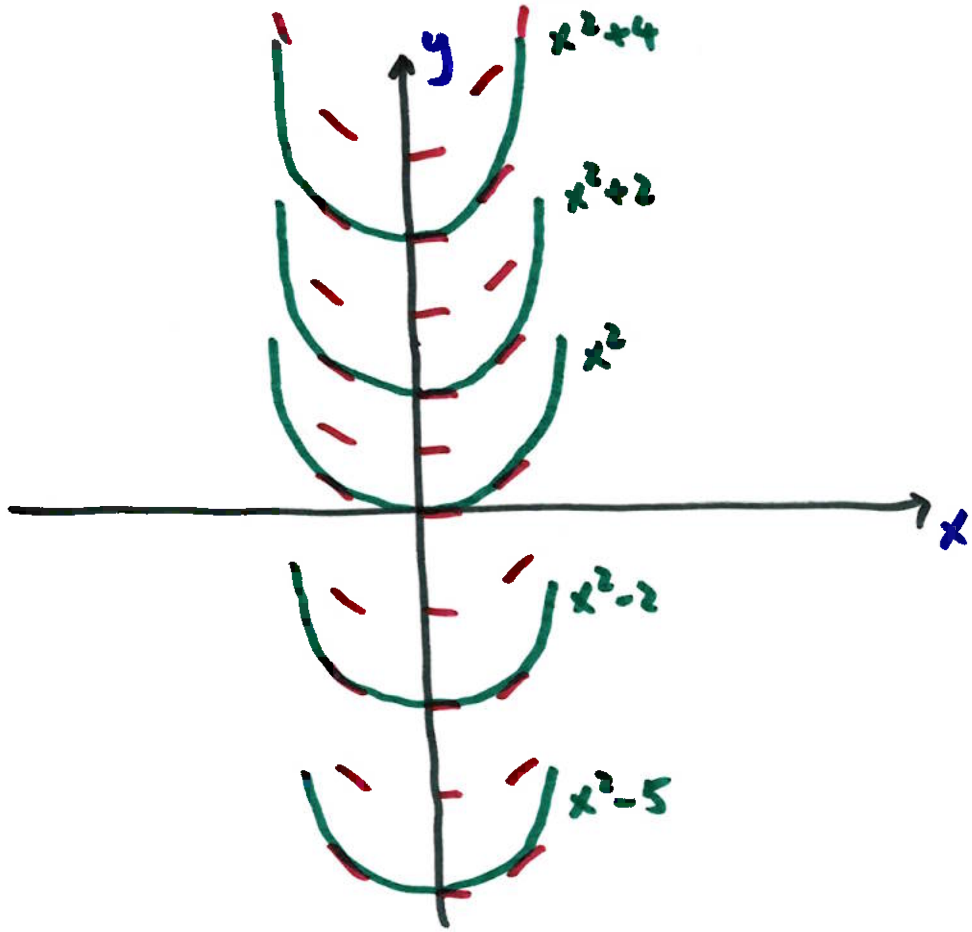
1.3 Slope Field and Solution Curves

a qualitative way to understand the ~~sto~~ solution

from last time: $\frac{dy}{dx} = 2x$ Solution: $y = x^2 + c$



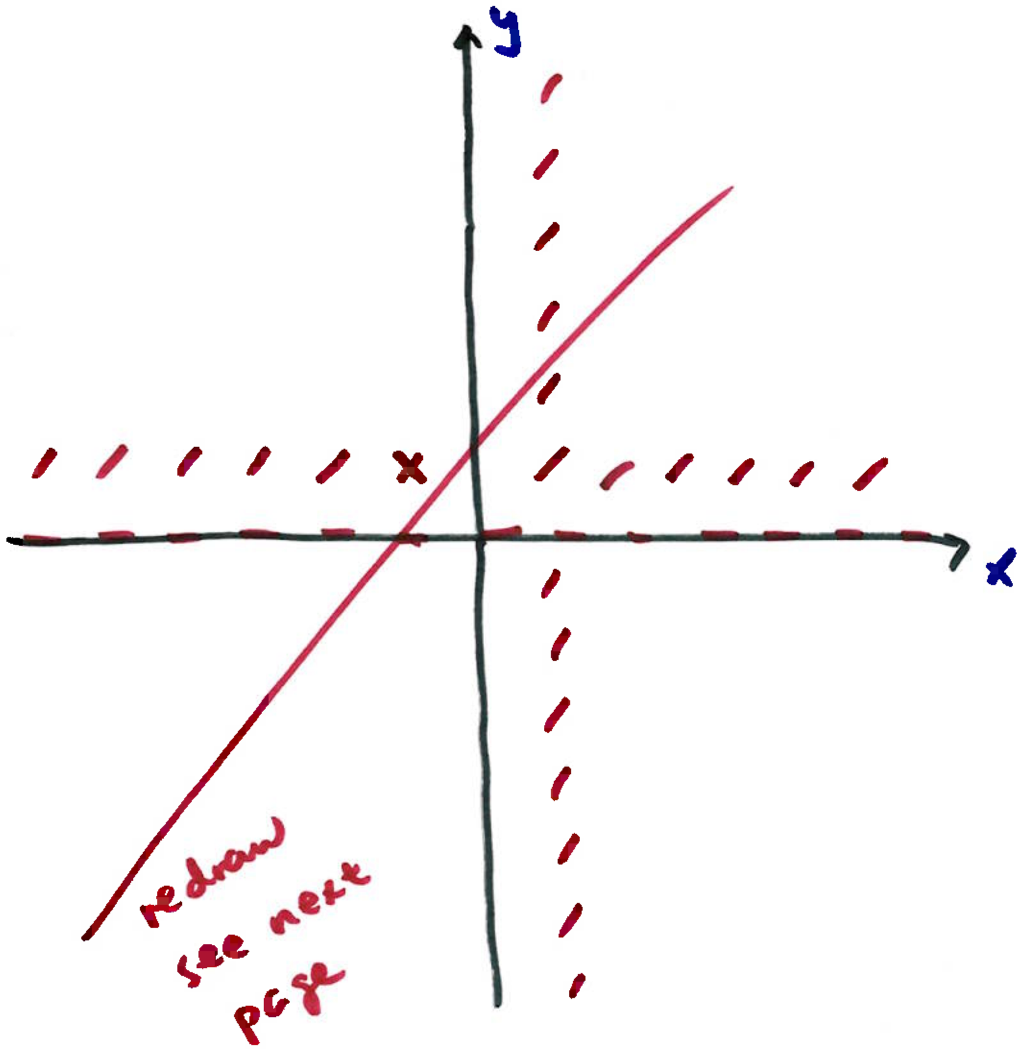
sketch lines tangent to curves at different points

each has slope given by the differential eq.

$$\frac{dy}{dx} = 2x$$

pretend we don't know solution is $x^2 + c$

construct a field of slopes using $\frac{dy}{dx} = 2x$

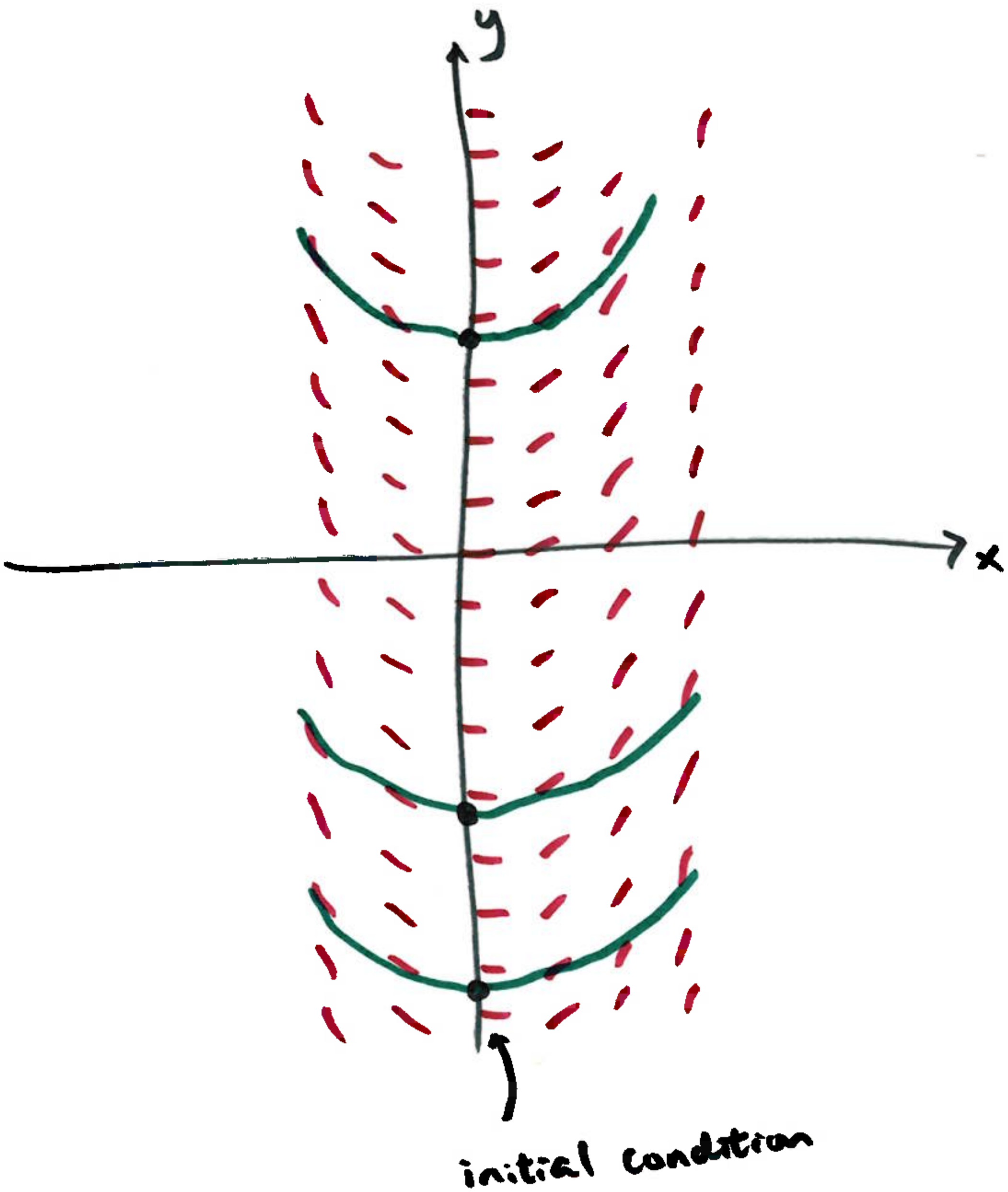


$\frac{dy}{dx} = 2x \rightarrow$ slope at point (x, y)

$(0, 0) \rightarrow$ slope is $2(0) = 0$

$(1, 1) \rightarrow$ slope is 2

note it does not depend on y: each ~~row~~ column all same slopes ~~(fix y)~~ fix x

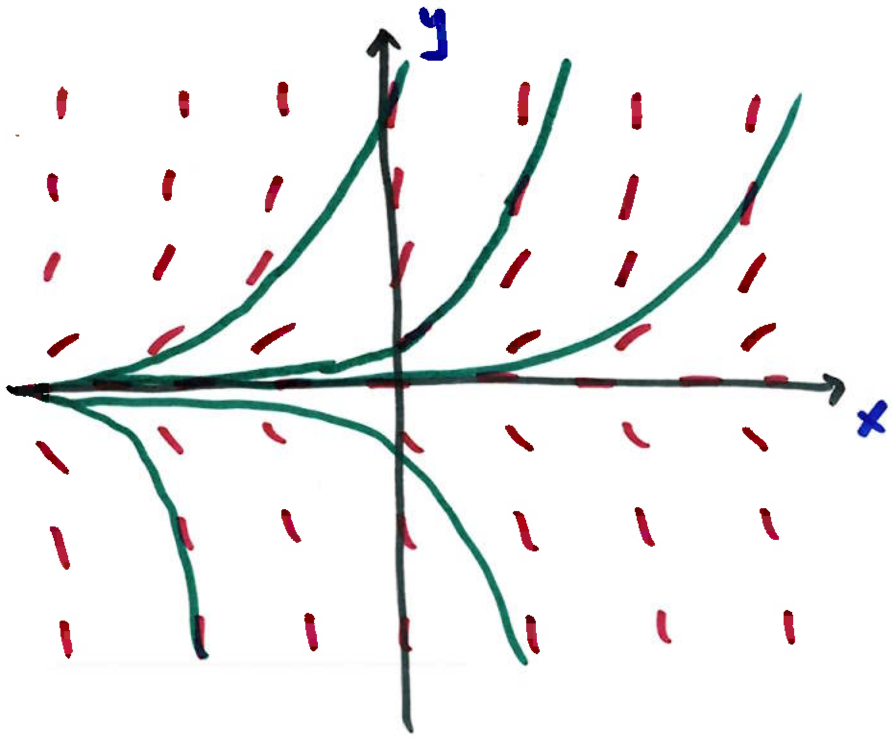


$$\frac{dy}{dx} = 2x$$

the solution curves can be seen in the slope field w/o having to solve for them

we can see that as $x \to \infty$, $y \to \infty$ and $x \to -\infty$, $y \to \infty$

example $y' = y$



this time slope depends on y only

each row all same slopes

$y=0: \ y'=0$

sketch a few solution curves using the slopes

even w/o solving, we see exponential behavior

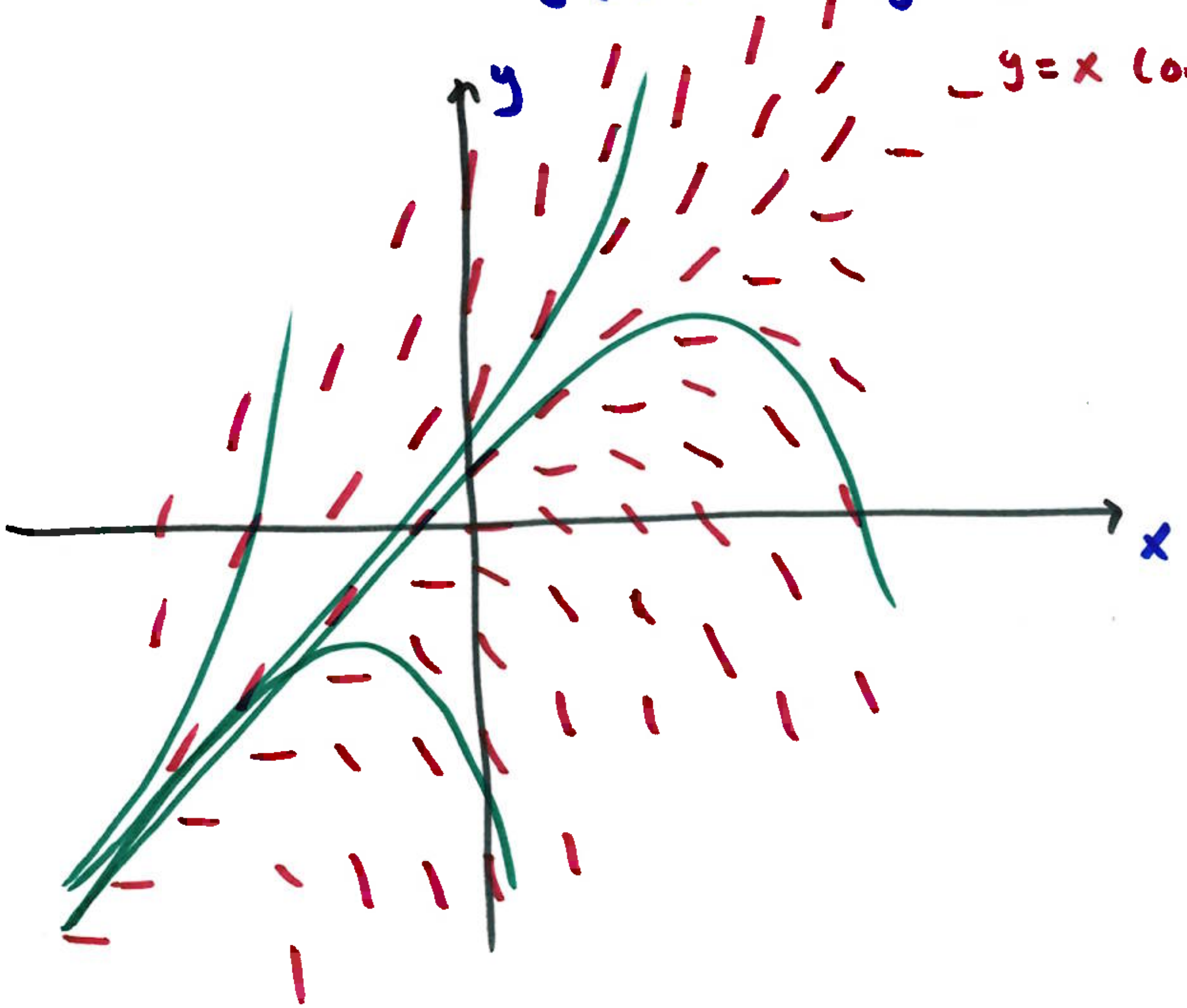
if $y(0) > 0 \rightarrow y \rightarrow \infty$

$y(0) < 0 \rightarrow y \rightarrow -\infty$

example $\dfrac{dy}{dx} = y - x$ slope at (x, y) is $y' = y - x$

notice if $y - x = 0 \rightarrow$ on the curve $y = x$

every point has $y' = 0$



$y = x$ (on here, $\dfrac{dy}{dx} = y - x = 0$)

above $y = x$

then $y - x > 0 \rightarrow y' > 0$

below $y = x \rightarrow y' < 0$

we can say:

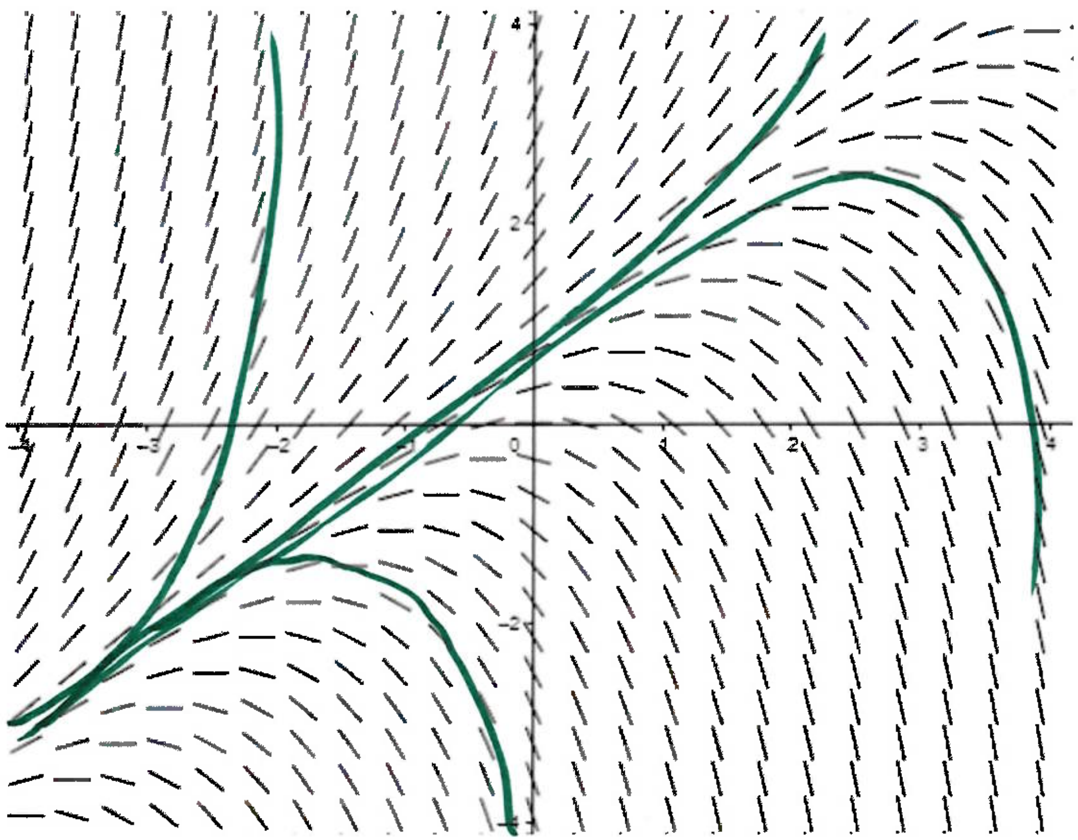
if $y(0)$ is above $y = x$

$y \rightarrow \infty$

if $y(0)$ is below $y = x$

$y \rightarrow -\infty$

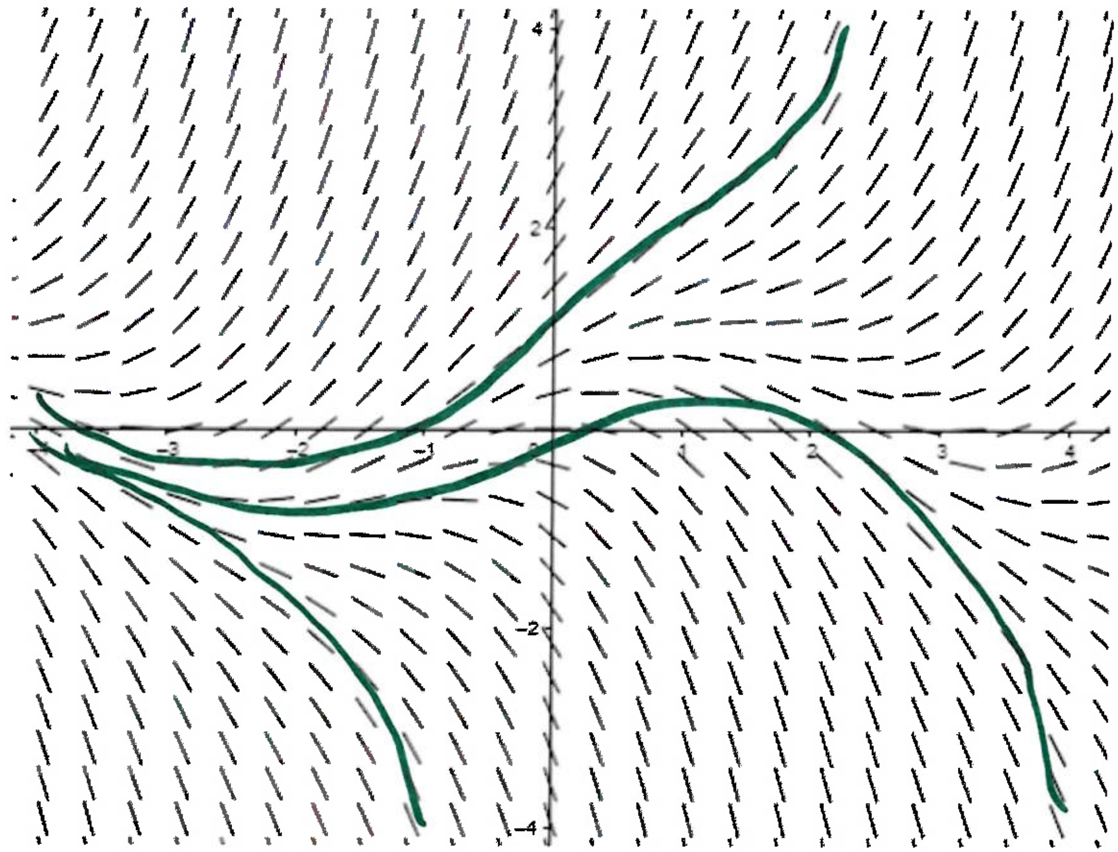
$$\frac{dy}{dx} = y - x$$



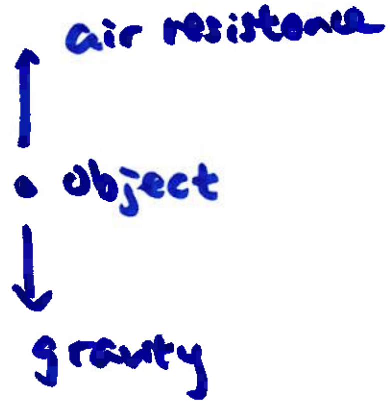
$\frac{dy}{dx} = y - \sin x$ → on $y = \sin x$ slope $= 0$

above $y = \sin x$ slope increases

below $y = \sin x$ " decreases



let's use the slope field to analyze the terminal velocity of a falling object



↓ velocity positive <u>down</u>

Newton's 2nd Law: $F = ma = m\dfrac{dv}{dt}$ v: velocity

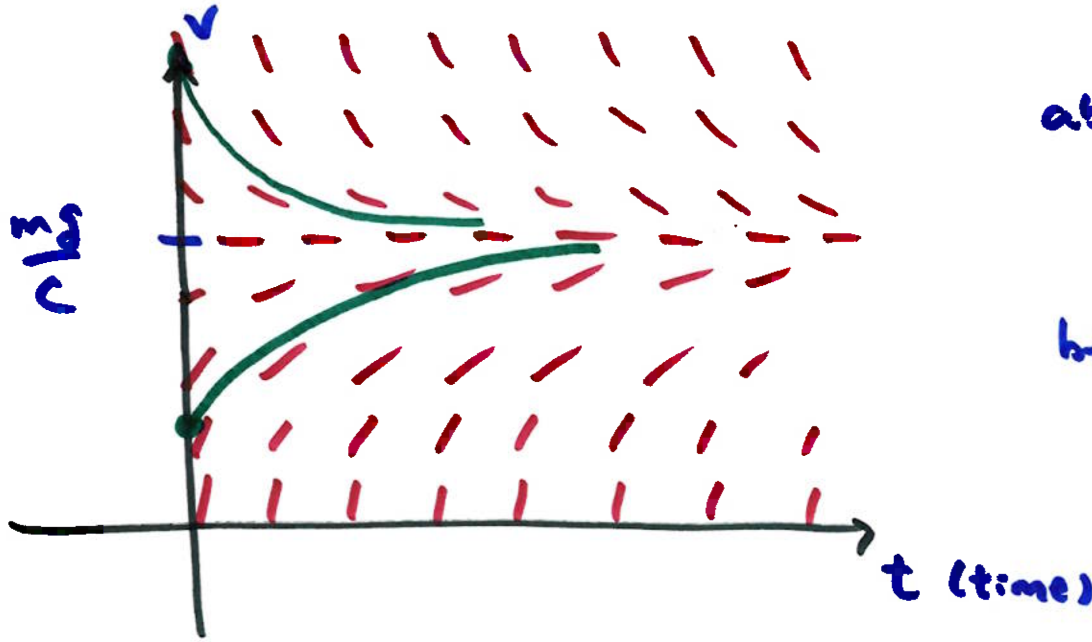
$F =$ gravity $-$ drag → can model as cv ← constant

$= mg - cv$

$$m\frac{dv}{dt} = mg - cv \rightarrow \boxed{\frac{dv}{dt} = g - \frac{cv}{m}}$$

$\frac{dv}{dt} = g - \frac{cv}{m}$ → on $g = \frac{cv}{m}$ slope is zero

or $v = \frac{mg}{c}$



above $v = \frac{mg}{c}$ slope is < 0

higher up → more negative

below $v = \frac{mg}{c}$: opposite

if $v(0) < \frac{mg}{c}$ → v increases ~~untttl~~ until $\frac{mg}{c} = v$ $\Big\}$ terminal velocity

$v(0) > \frac{mg}{c}$ → slow down and stay at $v = \frac{mg}{c}$

C is drag coefficient (parachute → high c so low $\frac{mg}{c}$)