

Remember that your work is graded on the quality of your writing and explanation as well as the validity of the mathematics.

- (1) (10 Points) Construct a 3×3 nonzero matrix A such that the vector $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ is a solution of $Ax = 0$.

Answer: For Example,

$$\begin{pmatrix} 2 & -1 & 0 \\ 3 & 0 & -1 \\ 1 & 1 & -1 \end{pmatrix}$$

- (2) (10 Points) Find the value(s) of h for which the vectors are linearly dependent. Justify each answer.

$$\begin{pmatrix} 1 \\ 5 \\ 3 \end{pmatrix}, \begin{pmatrix} 2 \\ 9 \\ 5 \end{pmatrix}, \begin{pmatrix} 4 \\ h \\ 9 \end{pmatrix}.$$

Answer: We can see that any two of these vectors are linearly independent, so we assume there exist weights a, b not all zero such that

$$a \begin{pmatrix} 1 \\ 5 \\ 3 \end{pmatrix} + b \begin{pmatrix} 2 \\ 9 \\ 5 \end{pmatrix} = \begin{pmatrix} 4 \\ h \\ 9 \end{pmatrix}$$

By solving the equations

$$\begin{cases} a + 2b = 4, \\ 3a + 5b = 9 \end{cases}$$

we have $a = -2, b = 3$, therefore we get

$$h = 5a + 9b = 17$$