Math 265 Summer 2018 - Quiz 03

22/6/2018 - 10 Minutes **Name:**

Remember that your work is graded on the quality of your writing and explanation as well as the validity of the mathematics.

(1) (10 Points) Show that $I_m A = A$ when A s an $m \times n$ matrix.

Proof:

Since for every $\mathbf{x} \in \mathbb{R}^m$, we have $I_m\mathbf{x} = \mathbf{x}$.

Assume that $A = [\mathbf{a_1}, \mathbf{a_2}, \cdots, \mathbf{a_n}]$, here $\mathbf{a}_i \in \mathbb{R}^m$, we get

$$I_m A = I_m[\mathbf{a_1}, \mathbf{a_2}, \cdots, \mathbf{a_n}] = [I_m\mathbf{a_1}, \cdots, I_m\mathbf{a_n}] = [\mathbf{a_1}, \mathbf{a_2}, \cdots, \mathbf{a_n}] = A$$

(2) (10 Points) Find the inverse of the matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 2 & 1 \\ 2 & 1 & 2 \end{bmatrix}$, if it exists.

Answer:

$$[A\ I] = \begin{bmatrix} 2 & 1 & 1 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 & 1 & 0 \\ 2 & 1 & 2 & 0 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 2 & 1 & 1 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{bmatrix}$$

$$\sim \begin{bmatrix} 2 & 1 & 0 & 2 & 0 & -1 \\ 0 & 2 & 0 & 1 & 1 & -1 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{bmatrix} \sim \begin{bmatrix} 2 & 0 & 0 & 3/2 & -1/2 & -1/2 \\ 0 & 2 & 0 & 1 & 1 & -1 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 0 & 0 & 3/4 & -1/4 & -1/4 \\ 0 & 1 & 0 & 1/2 & 1/2 & -1/2 \\ 0 & 0 & 1 & -1 & 0 & 1 \end{bmatrix}$$

Therefore

$$A^{-1} = \begin{bmatrix} 3/4 & -1/4 & -1/4 \\ 1/2 & 1/2 & -1/2 \\ -1 & 0 & 1 \end{bmatrix}$$