Math 265 Summer 2018 - Quiz 06

9/7/2018 - 10 Minutes **Name:**

Remember that your work is graded on the quality of your writing and explanation as well as the validity of the mathematics.

(1) (10 Points) For the following subspace, (a) find a basis, and (b) state the dimension.

$$A = \Big\{ \begin{bmatrix} -3a + b - 2c \\ a + 2c \\ 5b - c \\ -9a + 5b + 3c \end{bmatrix} \Big| a, b, c \in \mathbb{R} \Big\}$$

Answer:

(a) the basis is

$$\begin{bmatrix} -3 \\ 1 \\ 0 \\ -9 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 5 \\ 5 \end{bmatrix}, \begin{bmatrix} -2 \\ 2 \\ -1 \\ 3 \end{bmatrix}$$

(b)

$$\dim A = 3.$$

(2) (10 Points) The first four Hermite polynomials are $1, 2t, -2 + 4t^2$ and $-12t + 8t^3$. Show that the first four Hermite polynomials form a basis of $\mathbb{P}^3$.

Proof:

Let $f_0 = 1, f_1 = 2t, f_3 = -2 + 4t^2$ and $f_4 = -12t + 8t^3$, notice that all four polynomials have different degrees($\deg f_i = i$ for $i = 0, 1, 2, 3$).

Thus, we can say that f_1 doesn't belong to the space spanned by f_0.

Similarly, f_2 doesn't belong to the space spanned by f_0 and f_1.

Moreover, f_3 doesn't belong to the space spanned by f_0, f_1 and f_2.

Therefore, the first four Hermite polynomials are linearly independent and that the number of polynomials equals the dimension of $\mathbb{P}^3$. By the Basis Theorem, they form a basis of $\mathbb{P}^3$.