Math 265 Summer 2018 - Quiz 07

7/11/2018 - 10 Minutes **Name:**

Remember that your work is graded on the quality of your writing and explanation as well as the validity of the mathematics.

(1) (10 Points) Is $\lambda = 3$ an eigenvalue of $\begin{bmatrix} 4 & 2 & 3 \\ -1 & 1 & -3 \\ 2 & 4 & 9 \end{bmatrix}$? If so, find one corresponding eigenvector.

Answer: Assume vector $\mathbf{u} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$, we have

$$A\mathbf{u} = \begin{bmatrix} 4x_1 + 2x_2 + 3x_3 \\ -x_1 + x_2 - 3x_3 \\ 2x_1 + 4x_2 + 9x_3 \end{bmatrix}$$

Simplify the equations $A\mathbf{u} = 3\mathbf{u}$, we have $x_1 + 2x_2 + 3x_3 = 0$. Therefore, $A\mathbf{u} = 3\mathbf{u}$ has solutions of two free variables. $\lambda = 3$ is an eigenvalue of the given matrix.

Corresponding eigenvectors can be $\begin{bmatrix} 2 \\ -1 \\ 0 \end{bmatrix}$, or $\begin{bmatrix} 3 \\ 0 \\ -1 \end{bmatrix}$.

(2) (10 Points) Find the characteristic polynomial and all eigenvalues of matrix $\begin{bmatrix} 5 & -2 & 3 \\ 0 & 1 & 0 \\ 6 & 7 & -2 \end{bmatrix}$.

Answer: By caculating the determinant of $A - \lambda I = \begin{bmatrix} 5-\lambda & -2 & 3 \\ 0 & 1-\lambda & 0 \\ 6 & 7 & -2-\lambda \end{bmatrix}$ we get the characteristic polynomial of A should be

$$(1-\lambda)(\lambda^2 - 3\lambda - 28) = (1-\lambda)(\lambda - 7)(\lambda + 4)$$

Thus, all the eigenvalues are $\lambda = 1, 7, -4$.