Math 265 Summer 2018 - Quiz 08

7/17/2018 - 10 Minutes **Name:**

Remember that your work is graded on the quality of your writing and explanation as well as the validity of the mathematics.

(1) (10 Points) Diagonalize the following matrix, if possible.

$$\begin{bmatrix} 5 & -2 \\ 0 & 5 \end{bmatrix}$$

If not, explain why.

Answer: The eigenvalue of the gievn matrix is $\lambda = 5$, and the corresponding basis is $v = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$.

Since it only has one linearly independt eiegnvector, which means it doesn't satisfy the condition of the Diagonalization Theorem. Therefore, the gievn matrix is not diagonalizable.

(2) (10 Points) Diagonalize the following matrix, if possible.

$$A = \begin{bmatrix} 0 & -4 & -6 \\ -1 & 0 & -3 \\ 1 & 2 & 5 \end{bmatrix}$$

That is, find an invertible matrix P and a diagonal matrix D such that $A = PDP^{-1}$.

[Hint: the characteristic equation of given matrix can be factored: $\det(A - \lambda I) = -(\lambda - 1)(\lambda - 2)^2$]

Answer: The eigenvalues are $\lambda = 1$ and $\lambda = 2$.

We find the basis for each eigenspace:

- Basis for $\lambda = 1$: $v_1 = \begin{bmatrix} -2 \\ -1 \\ 1 \end{bmatrix}$;
- Basis for $\lambda = 2$: $v_2 = \begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}$, $v_3 = \begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix}$;

Now, we can construct P from the vectors:

$$P = [v_1 \ v_2 \ v_3] = \begin{bmatrix} -2 & -2 & -3 \\ -1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

and the diagonal matrix D as

$$D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$