**Math 265 Summer 2018 - Quiz 11**

**7/27/2018 - 10 Minutes** **Name:**

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Remember that your work is graded on the quality of your writing and explanation as well as the validity of the mathematics.

(1) (10 Points) The given set is a basis for a subspace $W$, use the Gram-Schmidt process to produce an orthogonal basis for $W$.

$$\mathbf{x_1} = \begin{bmatrix} 1 \\ 1 \\ -1 \\ 0 \end{bmatrix}, \mathbf{x_2} = \begin{bmatrix} 3 \\ -1 \\ -1 \\ 2 \end{bmatrix}, \mathbf{x_3} = \begin{bmatrix} 1 \\ 2 \\ 0 \\ -2 \end{bmatrix}$$

Here from the first two vectors $\mathbf{x_1}, \mathbf{x_2}$, we already have

$$\mathbf{v_1} = \begin{bmatrix} 1 \\ 1 \\ -1 \\ 0 \end{bmatrix}, \quad \mathbf{v_2} = \begin{bmatrix} 2 \\ -2 \\ 0 \\ 2 \end{bmatrix}$$

Find the third vector of the basis.

**Answer:**

$$\mathbf{v_3} = \mathbf{x_3} - \frac{\mathbf{x_3} \cdot \mathbf{v_1}}{\mathbf{v_1} \cdot \mathbf{v_1}}\mathbf{v_1} - \frac{\mathbf{x_3} \cdot \mathbf{v_2}}{\mathbf{v_2} \cdot \mathbf{v_2}}\mathbf{v_2} = \begin{bmatrix} 1 \\ 2 \\ 0 \\ -2 \end{bmatrix} - \frac{3}{3}\begin{bmatrix} 1 \\ 1 \\ -1 \\ 0 \end{bmatrix} - \frac{-6}{12}\begin{bmatrix} 2 \\ -2 \\ 0 \\ 2 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 1 \\ -1 \end{bmatrix}$$

(2) (10 Points) find a least-squares solution of $A\mathbf{x} = \mathbf{b}$.

$$A = \begin{bmatrix} 3 & 2 \\ 1 & 1 \\ 1 & 0 \end{bmatrix}, \quad \mathbf{b} = \begin{bmatrix} 2 \\ 0 \\ 5 \end{bmatrix}$$

**Answer:** We solve the equation

$$A^T A\hat{\mathbf{x}} = A^T\mathbf{b}$$

$$\begin{bmatrix} 3 & 1 & 1 \\ 2 & 1 & 0 \end{bmatrix} \begin{bmatrix} 3 & 2 \\ 1 & 1 \\ 1 & 0 \end{bmatrix} \hat{\mathbf{x}} = \begin{bmatrix} 3 & 1 & 1 \\ 2 & 1 & 0 \end{bmatrix} \begin{bmatrix} 2 \\ 0 \\ 5 \end{bmatrix}$$

$$\begin{bmatrix} 11 & 7 \\ 7 & 5 \end{bmatrix} \hat{\mathbf{x}} = \begin{bmatrix} 11 \\ 4 \end{bmatrix}$$

Therefore we get the least-square solution

$$\hat{\mathbf{x}} = \begin{bmatrix} 9/2 \\ -11/2 \end{bmatrix}$$