Math 265 Summer 2018 - Quiz 04

28/6/2018 - 10 Minutes **Name:**

Remember that your work is graded on the quality of your writing and explanation as well as the validity of the mathematics.

(1) (10 Points) Following is a matrix A and an echelon form of A, find bases for $NulA$, and state the rank of A.

$$A = \begin{bmatrix} 1 & -3 & 2 \\ -3 & 9 & -1 \\ 2 & -6 & 4 \end{bmatrix} \sim \begin{bmatrix} 1 & -3 & 2 \\ 0 & 0 & 5 \\ 0 & 0 & 0 \end{bmatrix} = B$$

Answer:

By solving Ax = 0, we get the bases for $NulA$:

$$\begin{bmatrix} 3 \\ 1 \\ 0 \end{bmatrix}.$$

$$Rank(A) = Rank(B) = 2.$$

(2) (10 Points) the vector $\mathbf{x}$ is in a subspace H with a basis $\mathcal{B} = \{\mathbf{x_1}, \mathbf{x_2}\}$, Find the $\mathcal{B}$-coordinate vector of $\mathbf{x}$.

$$\mathbf{b_1} = \begin{bmatrix} 1 \\ -3 \end{bmatrix}, \ \mathbf{b_2} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}, \ \mathbf{x} = \begin{bmatrix} 5 \\ 5 \end{bmatrix},$$

Answer: for the $\mathcal{B}$-coordinate vector

$$[\mathbf{x}]_{\mathcal{B}} = \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$$

We have

$$\mathbf{x} = c_1\mathbf{b_1} + c_2\mathbf{b_2}, \Rightarrow \begin{bmatrix} 5 \\ 5 \end{bmatrix} = c_1 \begin{bmatrix} 1 \\ -3 \end{bmatrix} + c_2 \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

by solving the above equation, we get $c_1 = 1, c_2 = 2$.