6.3 Predators and Competitors (part 2)

Competitors: going after the same resource but otherwise leave each other alone (e.g. chipmunks and squirrels)

$x(t)$, $y(t)$: competitor pops.

$$\frac{dx}{dt} = a_1 x - b_1 x^2 - c_1 xy = x(a_1 - b_1 x - c_1 y)$$

$$\frac{dy}{dt} = a_2 y - b_2 y^2 - c_2 xy = y(a_2 - b_2 y - c_2 x)$$

$$\boxed{a_i, b_i, c_i > 0}$$

if no interaction ($c_i = 0$)

$$\frac{dx}{dt} = a_1 x - b_1 x^2 = x(a_1 - b_1 x)$$ logistic growth cap at $x = \frac{a_1}{b_1}$

$$\frac{dy}{dt} = a_2 y - b_2 y^2 = y(a_2 - b_2 y)$$ cap at $y = \frac{a_2}{b_2}$

$c_i > 0$, then further reduction due to the other competitor cp's: $\underbrace{(0,0)}_{\text{both die}}$, $\underbrace{(0, a_2/b_2)}_{\text{one dies the other at cap}}$, $\underbrace{(a_1/b_1, 0)}$, $\underbrace{(x_c, y_c)}_{\text{coexistence}}$

$$\frac{dx}{dt} = x(a_1 - b_1 x - c_1 y) = x\left[a_1 - (b_1 x + c_1 y)\right]$$

$$\frac{dy}{dt} = \qquad = y\left[a_2 - \underbrace{(b_2 y + c_2 x)}\right]$$

reduction applied to exponential growth from environment and the other species

if $\frac{b_1}{c_1} > 1 \rightarrow x$ constrained <u>more</u> by environment than y

likewise, $\frac{b_2}{c_2} > 1 \rightarrow y$ " " " " x

$\frac{b_1}{c_1} \cdot \frac{b_2}{c_2} > 1 \rightarrow \boxed{b_1 b_2 > c_1 c_2}$ cap more important than competition

conversely, if $\boxed{b_1 b_2 < c_1 c_2}$ competition more important (at least one more likely to die) example

$$\frac{dx}{dt} = 60x - 3x^2 - 4xy \qquad b_1 = 3 \quad c_1 = 4$$

$$\frac{dy}{dt} = 42y - 3y^2 - 2xy \qquad b_2 = 3 \quad c_2 = 2$$

$b_1 b_2 > c_1 c_2$ competition less important

"weak" competition

coexistence likely

cp's: $(0,0)$, $(20,0)$, $(0,14)$, $(12,6)$

analyze stability about each

$$J = \begin{bmatrix} 60-6x-4y & -4x \\ -2y & 42-6y-2x \end{bmatrix}$$

$J(0,0) = \begin{bmatrix} 60 & 0 \\ 0 & 42 \end{bmatrix}$ λ's $> 0 \rightarrow$ Source unstable

$J(20,0) = \begin{bmatrix} -60 & -80 \\ 0 & 2 \end{bmatrix}$ λ's oppo. signs $\rightarrow$ saddle unstable

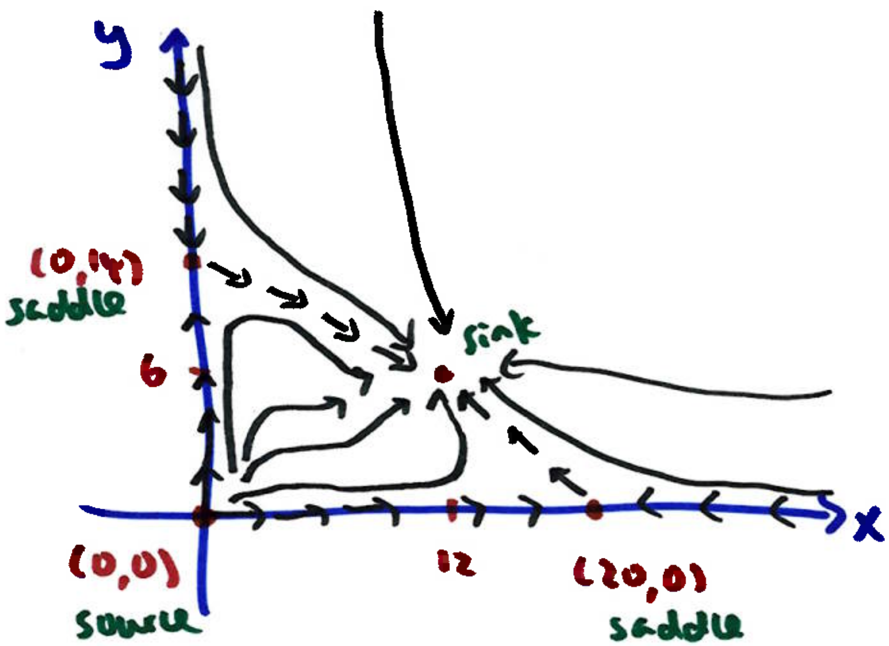
$$J(0,14) = \begin{bmatrix} 4 & 0 \\ -28 & -42 \end{bmatrix}$$ saddle unstable

$$J(12,6) = \begin{bmatrix} -36 & -48 \\ -12 & -18 \end{bmatrix}$$ λ's are both real and negative

sink asymptotically stable

so, we expect $(t \to \infty)$ coexistence

Sketch of phase diagram



example

$$\frac{dx}{dt} = 60x - 4x^2 - 3xy \qquad b_1 = 4 \qquad c_1 = 3$$

$$\frac{dy}{dt} = 42y - 2y^2 - 3xy \qquad b_2 = 2 \qquad c_2 = 3$$

$b_1 b_2 < c_1 c_2$ competition important

"strong" comp.

Somebody goes extinct

cp: $(0,0), (0,21), (15,0), (6,12)$

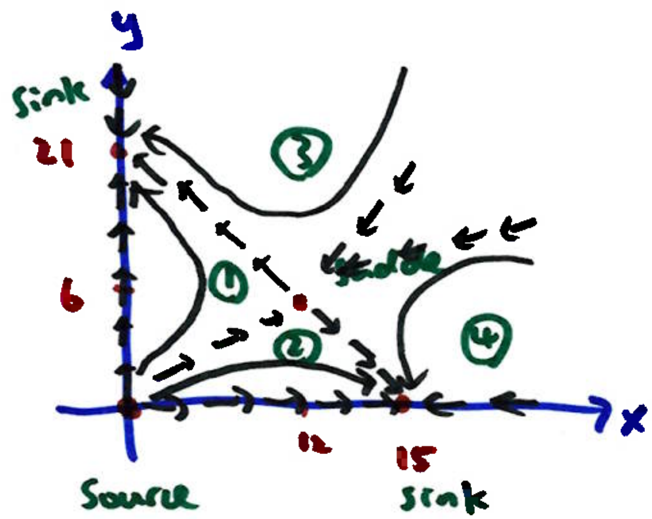
after analyzing the same way,

$(0,0)$: source unstable

$(0,21)$: ~~source~~ sink stable

$(15,0)$: sink stable

$(6,12)$: saddle unstable



Initial in ① $\to$ $x \to 0$

② $\to$ $y \to 0$

$$\frac{dx}{dt} = x(a_1 - b_1 x - c_1 y)$$

pos.

$$\frac{dy}{dt} = y(a_2 - b_2 y - c_2 x)$$

if $c_i < 0$ → add to growth rate

→ species in <u>cooperation</u>

(ants and aphids)

$$\frac{dx}{dt} = 30x - 3x^2 + xy$$

$$\frac{dy}{dt} = 60y - 3y^2 + 4xy$$

w/o interaction: $(0,0)$, $(10,0)$

$(0,20)$, $(10,20)$

w/ interaction: $(0,0)$, $(10,0)$

$(0,20)$, $(30,60)$

sink

$x \to 30$ which is higher than its cap at $x = 10$

No interaction