Recap/ Look ahead

$f(t)$ period $2L$ $\quad -L < t < L$

$$f(t) \sim \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi t}{L}\right) + b_n \sin\left(\frac{n\pi t}{L}\right)$$

$$a_n = \frac{1}{L}\int_{-L}^{L} f(t)\cos\left(\frac{n\pi t}{L}\right)dt \qquad b_n = \frac{1}{L}\int_{-L}^{L} f(t)\sin\left(\frac{n\pi t}{L}\right)dt$$

$$a_n = \frac{1}{L}\int_{0}^{2L} f(t)\cos\left(\frac{n\pi t}{L}\right)dt \qquad b_n = \frac{1}{L}\int_{0}^{2L} f(t)\sin\left(\frac{n\pi t}{L}\right)dt$$

Sometimes, we would rather work with the period $P = 2L$ instead of half period L

replace all L's above with $\frac{P}{2}$

$$f(t) \sim \frac{1}{2}a_0 + \sum_{n=1}^{\infty} a_n \cos\left(\frac{2n\pi t}{P}\right) + b_n \sin\left(\frac{2n\pi t}{P}\right)$$

$$a_n = \frac{2}{P}\int_{0}^{P} f(t)\cos\left(\frac{2n\pi t}{P}\right)dt \qquad b_n = \frac{2}{P}\int_{0}^{P} f(t)\sin\left(\frac{2n\pi t}{P}\right)dt$$

Fourier series extracts different frequencies out of a periodic $f(t)$

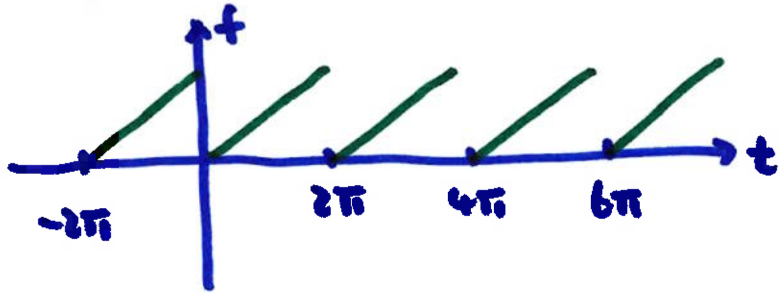
like $e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \cdots$

e^x is made of parts of x^n

the biggest coefficients are the dominant ones,

e^x is mostly made of 1 and x (especially good if x is near 0)

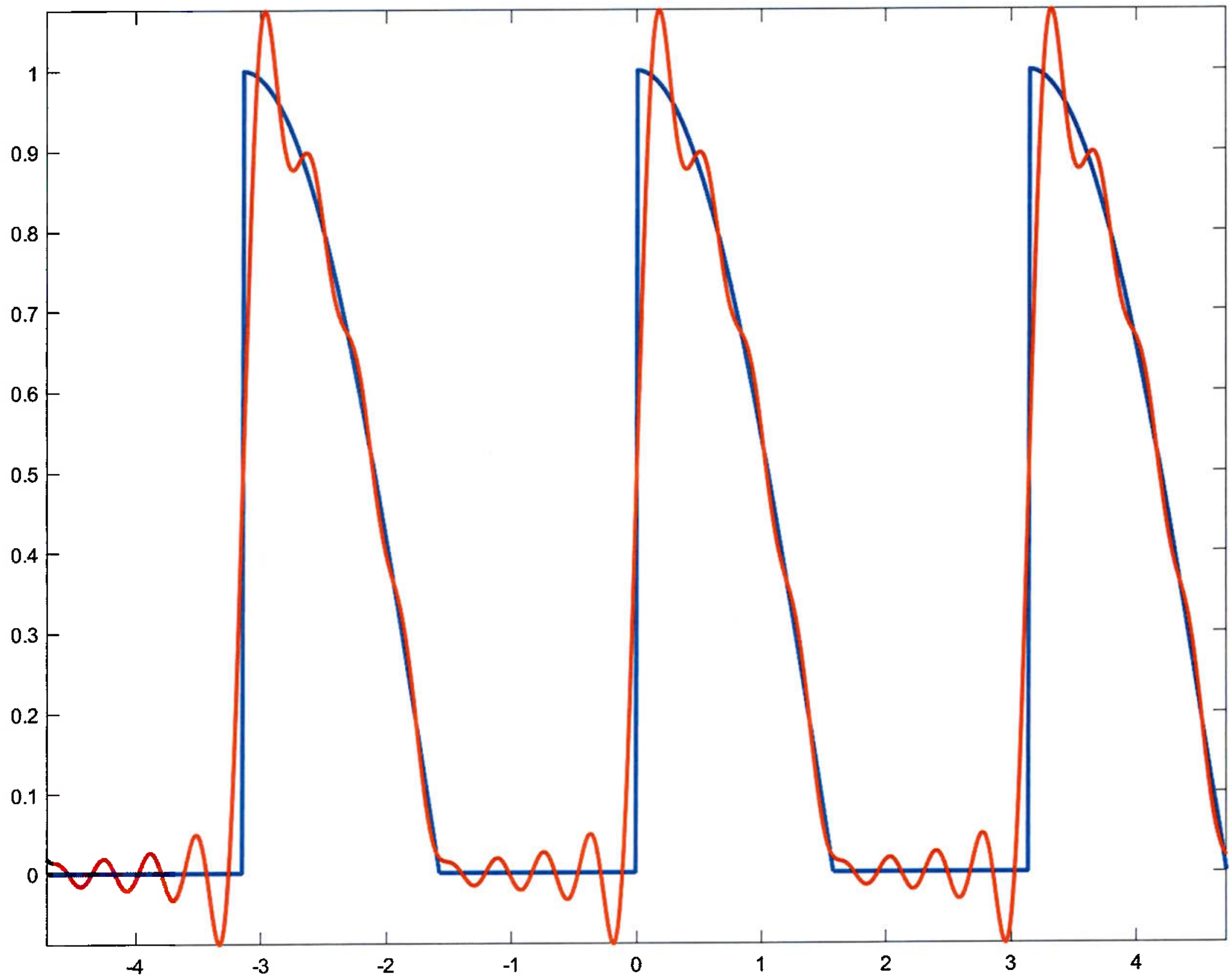
now, $f(t) = t$, period 2π $\quad 0<t<2\pi$



we find $a_0 = \pi$, $b_n = -\frac{2}{n}$, other $a_n = 0$

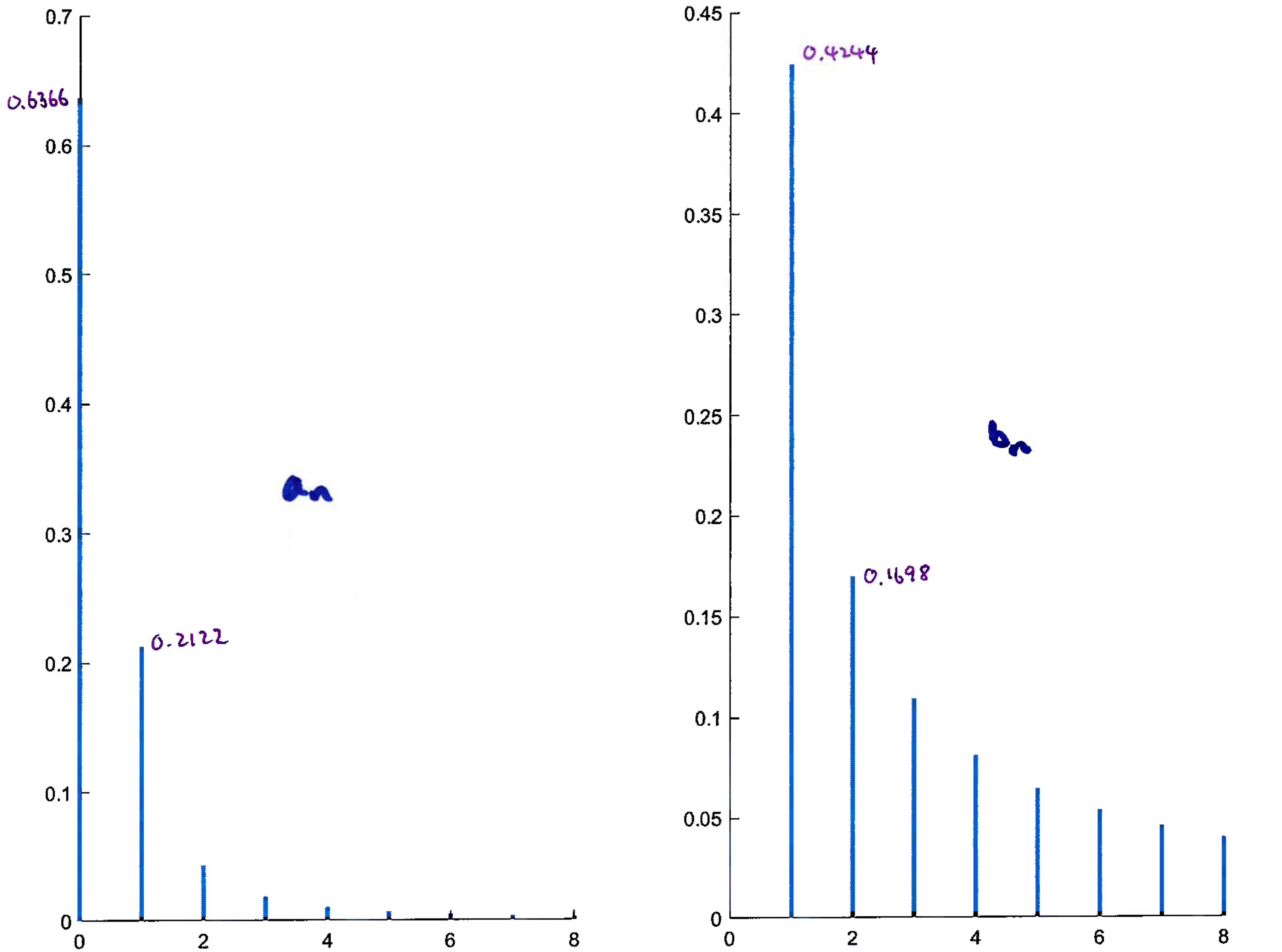
$$t \sim \frac{\pi}{2} - 2\sin(t) - \sin(2t) - \frac{2}{3}\sin(3t) - \frac{1}{2}\sin(4t) - \frac{2}{5}\sin(5t) - \cdots$$

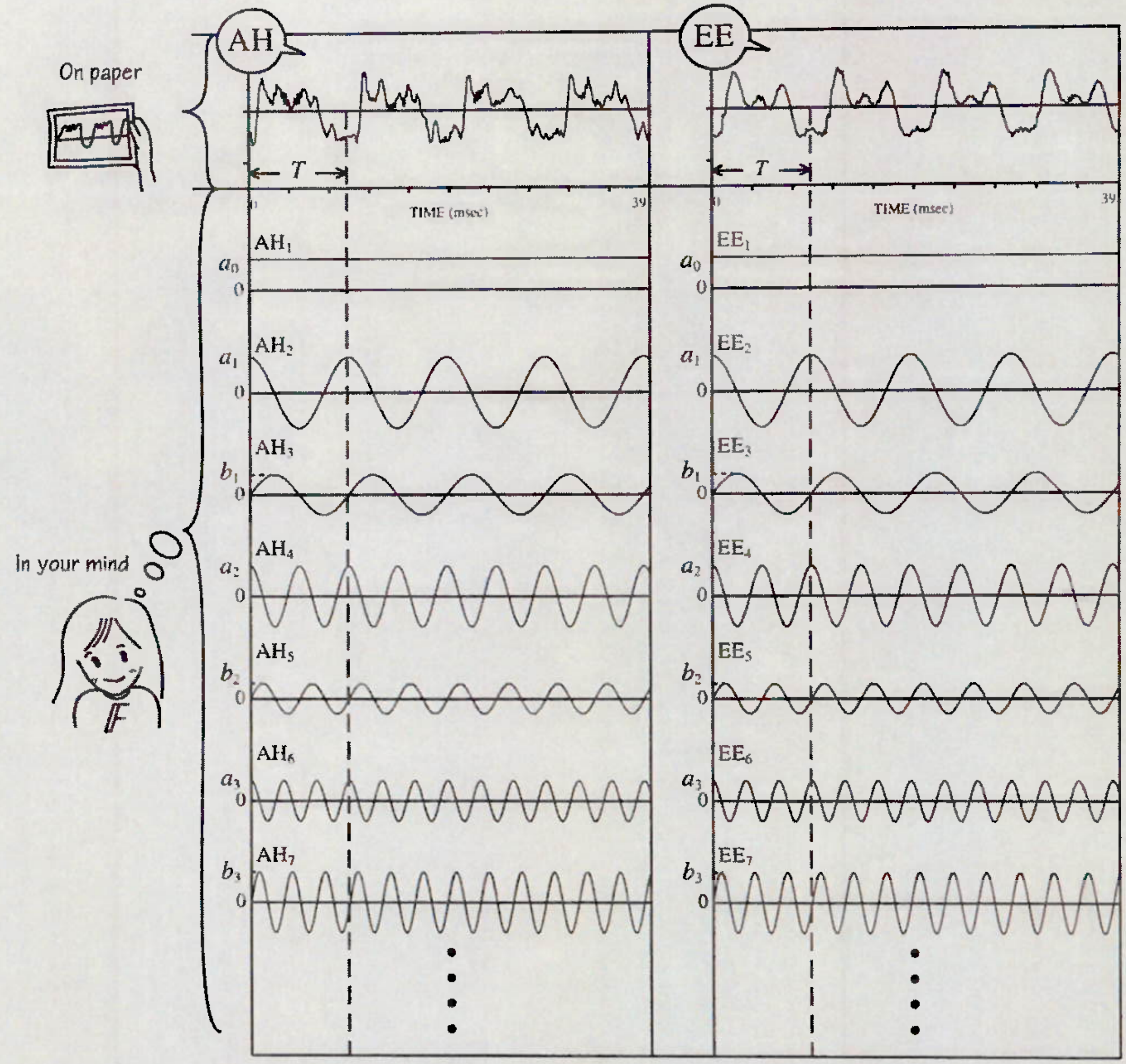
most explained by constant, $\sin(t)$, $\sin(2t)$, $\sin(3t)$



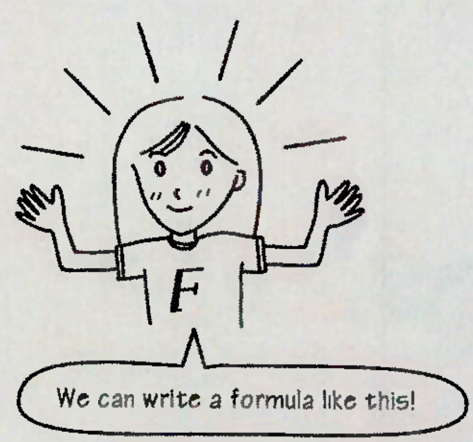
1
0.9
0.8
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
-4
-3
-2
-1
0
1
2
3
4

magnitudes of a_n, b_n



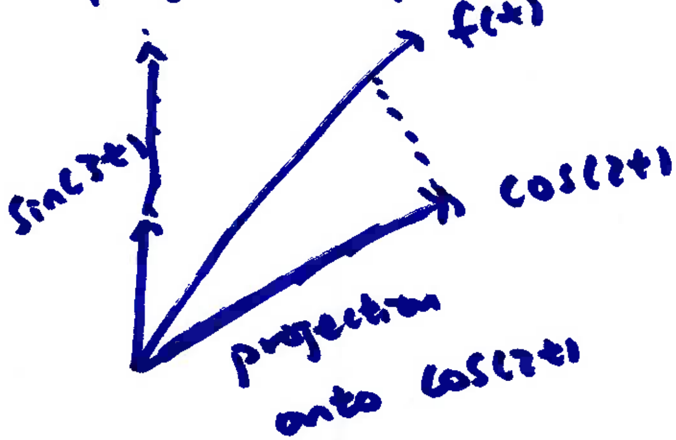


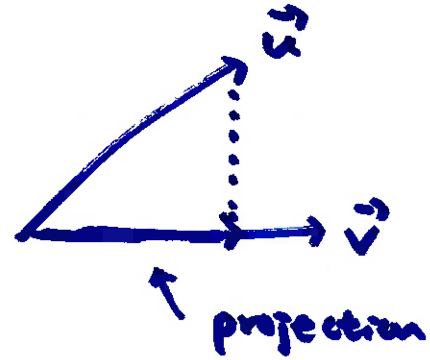
AH wave = $AH_1 + AH_2 + AH_3 + AH_4 + \ldots$ EE wave = $EE_1 + EE_2 + EE_3 + EE_4 + \ldots$



$$\frac{1}{L}\int_{-L}^{L} f(t)\cos\left(\frac{n\pi t}{L}\right)dt$$ extracts part of $f(t)$ explained by $\cos\left(\frac{n\pi t}{L}\right)$

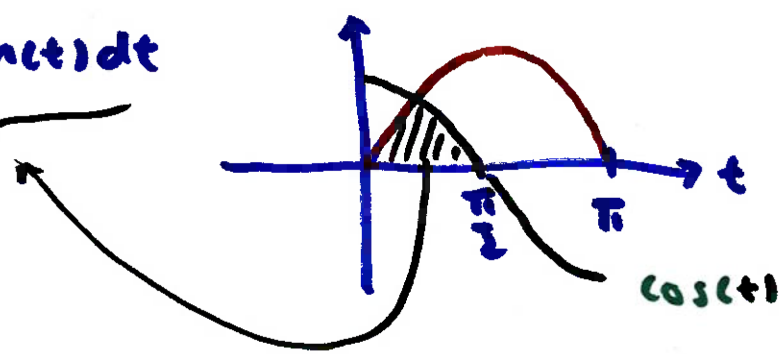
projection (generalized)



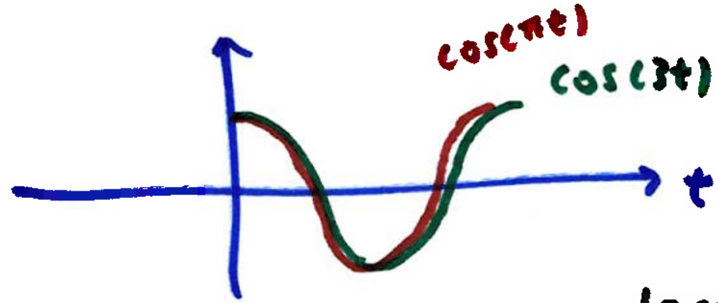


it is also some measure of the overlap between $f(t)$ and $\cos\left(\frac{n\pi t}{L}\right)$

for example, $\displaystyle\int_0^{\pi} \underbrace{\cos(t)\sin(t)\,dt}$



$$\int_0^{2\pi} \cos(3t)\cos(\pi t)\,dt$$



lots of overlap → large integral value

Look ahead

rest of ch.9 we will encounter these equations A LOT

$x'' + \lambda x = 0$ $\quad$ $x(0) = x(L) = 0$ $\quad$ boundary-value prob

or $x'(0) = x'(L) = 0$

or some mix

Solution: characteristic eq $\quad r^2 + \lambda = 0$

if $\lambda = 0$, $r = 0$ $\quad$ polynomial

$\lambda < 0$, $\quad$ exponentials

$\lambda > 0$, $\quad$ sine/cosine

only way to meet $x(0) = x(L) = 0$

$x' + ax = 0$ 1st-order char. eq. $r + a = 0$ $r = -a$

solution: $x = Ce^{rt} = Ce^{-at}$

e^{-at} (w/o constant) is fundamental solution