Complex eigenthings

$$\vec{x}' = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix} \vec{x}$$

solution: $\vec{x}(t) = c_1 \underbrace{e^{\lambda_1 t}\vec{v}_1}_{\vec{x}_1} + c_2 \underbrace{e^{\lambda_2 t}\vec{v}_2}_{\vec{x}_2}$

$\det(A-\lambda I)=0$

$$\begin{vmatrix} 1-\lambda & 2 \\ -2 & 1-\lambda \end{vmatrix} = 0$$

$(1-\lambda)^2+4=0$ $\qquad 1-\lambda = \pm 2i$

$\lambda_1 = 1+2i, \qquad \lambda_2 = 1-2i$

$\lambda_1 = 1+2i$

$(A-\lambda_1 I)\vec{v}_1 = \vec{0}$

$-2i \cdot i = -2i^2 = -2 \cdot -1 = 2$

$$\left[\begin{array}{cc|c} -2i & 2 & 0 \\ -2 & -2i & 0 \end{array}\right] \underset{}{\overset{\text{mult. R1 by } i \text{, add to R2}}{\sim}} \left[\begin{array}{cc|c} -2i & 2 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

$$\left[\begin{array}{cc|c} i & -1 & 0 \\ 0 & 0 & 0 \end{array}\right] \qquad \vec{v}_1 = \begin{bmatrix} a \\ b \end{bmatrix} \qquad b = r$$

$R1: \ ia - b = 0$

$ia = b \qquad a = \frac{b}{i} = \frac{r}{i}$

$r = i \rightarrow a = 1, \ b = i$

$$\boxed{\vec{v}_1 = \begin{bmatrix} 1 \\ i \end{bmatrix} \qquad \lambda_1 = 1 + 2i}$$

$\underline{\lambda_2 = 1 - 2i}$

$(A - \lambda_2 I)\vec{v}_2 = \vec{0}$

$$\left[\begin{array}{cc|c} 2i & 2 & 0 \\ -2 & 2i & 0 \end{array}\right] \sim \left[\begin{array}{cc|c} 2i & 2 & 0 \\ 0 & 0 & 0 \end{array}\right] \sim \left[\begin{array}{cc|c} i & 1 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

$$\boxed{\vec{v}_2 = \begin{bmatrix} 1 \\ -i \end{bmatrix} \qquad \lambda_2 = 1 - 2i}$$

$$\vec{x}_1 = e^{\lambda_1 t}\vec{v}_1 = e^{(1+2i)t}\begin{bmatrix} 1 \\ i \end{bmatrix} = e^{t} e^{i(2t)}\begin{bmatrix} 1 \\ i \end{bmatrix}$$

$$e^{ix} = \cos(x) + i\sin(x)$$

$$= e^{t}\left(\cos(2t) + i\sin(2t)\right)\begin{bmatrix} 1 \\ i \end{bmatrix}$$

$$\vec{x}_1$$

$$= e^{t}\begin{bmatrix} \cos(2t) + i\sin(2t) \\ -\sin(2t) + i\cos(2t) \end{bmatrix} = \boxed{e^{t}\begin{bmatrix} \cos(2t) \\ -\sin(2t) \end{bmatrix} + i e^{t}\begin{bmatrix} \sin(2t) \\ \cos(2t) \end{bmatrix}}$$

$$\vec{x}_2 = e^{\lambda_2 t}\vec{v}_2 = e^{(1-2i)t}\begin{bmatrix} 1 \\ -i \end{bmatrix}$$

$$e^{i(-2t)} = \cos(-2t) + i\sin(-2t)$$
$$= \cos(2t) - i\sin(2t)$$

$$= e^{t}\left(\cos(2t) - i\sin(2t)\right)\begin{bmatrix} 1 \\ -i \end{bmatrix}$$

$$\vec{x}_2$$

$$= e^{t}\begin{bmatrix} \cos(2t) - i\sin(2t) \\ -\sin(2t) - i\cos(2t) \end{bmatrix} = \boxed{e^{t}\begin{bmatrix} \cos(2t) \\ -\sin(2t) \end{bmatrix} - i e^{t}\begin{bmatrix} \sin(2t) \\ \cos(2t) \end{bmatrix}}$$

gen. solution: $\vec{x}(t) = A\vec{x}_1 + B\vec{x}_2$

complex pairs too

$$\vec{x}(t) = Ae^t \begin{bmatrix} \cos(2t) \\ -\sin(2t) \end{bmatrix} + iAe^t \begin{bmatrix} \sin(2t) \\ \cos(2t) \end{bmatrix}$$

$$+ Be^t \begin{bmatrix} \cos(2t) \\ -\sin(2t) \end{bmatrix} - iBe^t \begin{bmatrix} \sin(2t) \\ \cos(2t) \end{bmatrix}$$

$$= \overset{C_1}{\underbrace{(A+B)}_{\text{real}}} e^t \begin{bmatrix} \cos(2t) \\ -\sin(2t) \end{bmatrix} + \overbrace{\underbrace{i(A-B)}_{\text{real}}}^{C_2} \begin{bmatrix} \sin(2t) \\ \cos(2t) \end{bmatrix}$$

b/c complex conj. pairs

$$= C_1 e^t \begin{bmatrix} \cos(2t) \\ -\sin(2t) \end{bmatrix} + C_2 e^t \begin{bmatrix} \sin(2t) \\ \cos(2t) \end{bmatrix}$$

Application of stability

747 at 40,000 ft cruising altitude

$$\begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{q} \\ \dot{\theta} \end{bmatrix} = \underbrace{\begin{bmatrix} -0.003 & 0.039 & 0 & -0.322 \\ -0.065 & -0.319 & 7.74 & 0 \\ 0.020 & -0.101 & -0.429 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}}\begin{bmatrix} u+u_w \\ v+v_w \\ q \\ \theta \end{bmatrix} + \underset{\text{control}}{\vec{w}}$$

$q = \dot{\theta}$

Solution: $\begin{bmatrix} u(t) \\ v(t) \\ q(t) \\ \theta(t) \end{bmatrix} = \cdots$ stable?

$$\underbrace{\lambda = \overbrace{-0.375}^{\text{strong damp}} \pm \overbrace{0.8818}^{\text{high freq.}}\, i}_{}$$

asympt. stable!

Short period oscillation

period $\approx$ 7 seconds

decays in $\approx$ 10 seconds

no effect on altitude and speed

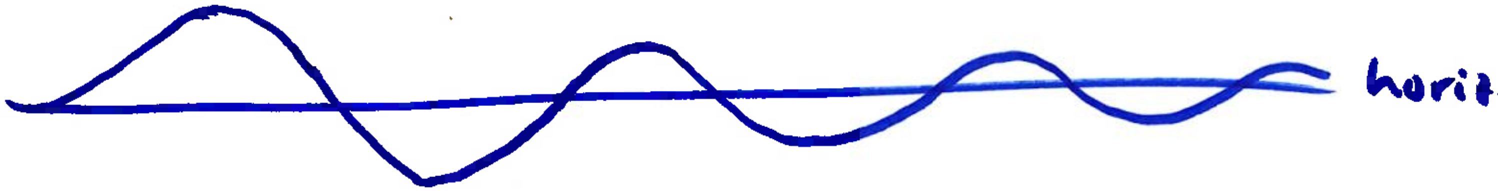
$$\underbrace{\lambda = \underbrace{-0.0005}_{\text{weak damping}} \pm \underbrace{0.0694}_{\text{low freq}}\, i}_{}$$

phugoid oscillation

period 93 sec

decays in 80 min

affects speed and altitude



Ch. 7 Laplace Transform

review: integration by parts $\rightarrow$ $\int f(x)dx = uv - \int v\,du$

improper integral $\rightarrow$ $\int_a^b f(x)dx$ $\quad a, b = \pm\infty$ or if $f(x)$ DNE $a < x < b$

$\int x e^x dx$ $\qquad$ (Laplace: $\int_0^\infty f(t) e^{-st} dt$)

order of picking u: LIATE

L: log
I: inv. trig
A: algebraic
T: trig
E: exp

$\int x e^x dx$

x: alg.
e^x: exp.

$u = x \qquad dv = e^x\,dx \qquad uv - \int v\,du = xe^x - \int e^x\,du$

$du = dx \qquad v = e^x \qquad = xe^x - e^x + C$

<u>Improper</u>

$$\int_0^\infty x e^{-x}\,dx = \lim_{a\to\infty} \int_0^a x e^{-x}\,dx$$

$u = x \qquad dv = e^{-x}\,dx$

$du = dx \qquad v = -e^{-x}$

$$= \lim_{a\to\infty} \left(-xe^{-x} \Big|_0^a + \int_0^a e^{-x}\,dx \right)$$

$$= \lim_{a\to\infty} \left(-ae^{-a} - e^{-x} \Big|_0^a \right)$$

$$= \lim_{a\to\infty} \left(\cancel{-ae^{-a}}^{\,0} - \cancel{e^{-a}}^{\,0} + 1 \right) = 1$$

is a number → integral converges