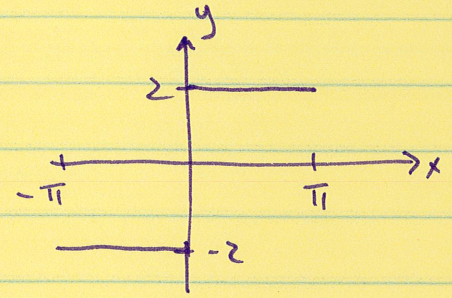
Quiz 10

1.



odd function, so sine series

$a_n = 0$ for all n

$$b_n = \frac{2}{L}\int_0^L f(x)\sin\frac{n\pi x}{L}\,dx \qquad L = \pi$$

$$b_n = \frac{2}{\pi}\int_0^{\pi} 2\sin nx\,dx = -\frac{4}{n\pi}\cos nx\Big|_0^{\pi}$$

$$= -\frac{4}{n\pi}\left(\cos n\pi - 1\right) \qquad n = 1, 2, 3, \ldots$$

$$= -\frac{4}{n\pi}\left[(-1)^n - 1\right] = \begin{cases} \frac{8}{n\pi} & \text{if } n \text{ is even} \\ 0 & \text{if } n \text{ is odd} \end{cases}$$

$$f(x) = \sum_{n=1}^{\infty} -\frac{4}{n\pi}\left(\cos n\pi - 1\right)\sin nx$$

or $$f(x) = \sum_{k=1}^{\infty} \frac{8}{(2k-1)\pi}\sin\left[(2k-1)x\right]$$

2. $X''T = XT'' \qquad u = XT$

$$\frac{X''}{X} = \frac{T''}{T} = -\lambda$$

$X(0) = 0, \quad X(L) = 0$

$u_t(x,0) = 0$ so $XT'(0) = 0 \rightarrow T'(0) = 0$

$X'' + \lambda X = 0 \qquad X(0) = X(L) = 0$

$$\lambda_n = \left(\frac{n\pi}{L}\right)^2 \qquad X_n = \sin\left(\frac{n\pi x}{L}\right)$$

$T'' + \lambda T = 0$

$T'' + \frac{n^2\pi^2}{L^2}T = 0$

$$T = k_1 \cos\frac{n\pi t}{L} + k_2 \sin\frac{n\pi t}{L} \qquad T'(0) = 0$$

$$T' = -k_1 \frac{n\pi}{L} \sin\frac{n\pi t}{L} + \frac{n\pi}{L} k_2 \cos\frac{n\pi t}{L}$$

$$0 = k_2$$

$$T_n = \cos\frac{n\pi t}{L}$$

$$u(x,t) = \sum_{n=1}^{\infty} C_n \cos\frac{n\pi t}{L} \sin\frac{n\pi x}{L}$$

~~$u(x,t) = \sum_{n=1}^{\infty} C_n$~~

$$u(x,0) = \sin\frac{\pi x}{L} + 0.5 \sin\frac{3\pi x}{L}$$

$$= \sum_{n=1}^{\infty} C_n \sin\frac{n\pi x}{L}$$

so $C_1 = 1$, $C_3 = 0.5$ and all others are zero

$$u(x,t) = \cos\frac{\pi t}{L} \sin\frac{\pi x}{L} + 0.5 \cos\frac{3\pi t}{L} \sin\frac{3\pi x}{L}$$