Quiz 1

1. $$\sum_{n=1}^{\infty} \frac{(-1)^n n^2 (x+2)^n}{3^n}$$

$$\lim_{n\to\infty} \left| \frac{(-1)^{n+1} (n+1)^2 (x+2)^{n+1}}{3^{n+1}} \cdot \frac{3^n}{(-1)^n n^2 (x+2)^n} \right| < 1$$

$$\lim_{n\to\infty} \left| (-1) \left(\frac{n+2}{n}\right)^2 \frac{1}{3} (x+2) \right| < 1$$

$$\left| \frac{x+2}{3} \right| < 1$$

$$-1 < \frac{x+2}{3} < 1$$

$$-3 < x+2 < 3$$

$$-5 < x < 1$$

$\boxed{\text{radius of conv. is } 3}$

2. $y'' - xy' = 0 \quad x_0 = 0 \qquad y = \sum_{n=0}^{\infty} a_n x^n \quad y' = \sum_{n=1}^{\infty} a_n n x^{n-1}$

$$y'' = \sum_{n=2}^{\infty} a_n (n)(n-1) x^{n-2}$$

$$\sum_{n=2}^{\infty} a_n (n)(n-1) x^{n-2} - \sum_{n=1}^{\infty} a_n (n) x^n = 0$$

$$\sum_{n=0}^{\infty} a_{n+2} (n+2)(n+1) x^n - \sum_{n=1}^{\infty} a_n (n) x^n = 0$$

$n=0$: $\quad 2a_2 = 0 \rightarrow a_2 = 0$

$n \geq 1$: $\quad a_{n+2}(n+2)(n+1) - a_n(n) = 0$

$$\boxed{a_{n+2} = \frac{n\, a_n}{(n+2)(n+1)} \qquad n = 1, 2, 3, 4 \ldots}$$

recurrence relation

indices differ by two, so separate into evens and odds

n = even:

$n=2$: $a_4 = \frac{2}{(4)(3)} a_2 = 0$ $(a_2 = 0)$

$n=4$: $a_6 = \frac{4}{(6)(5)} a_4 = 0$

in fact, $a_n = 0$ if n is even

n = odd

$n=1$: $a_3 = \frac{1}{3 \cdot 2} a_1 = \frac{1}{6} a_1$

$n=3$: $a_5 = \frac{3}{5 \cdot 4} a_3 = \frac{3}{5 \cdot 4} \cdot \frac{1}{6} a_1 = \frac{1}{40} a_1$

$n=5$: $a_7 = \frac{5}{7 \cdot 6} a_5 = \frac{5}{7 \cdot 6} \cdot \frac{1}{40} a_1 = \frac{1}{336} a_1$

Solution: $y = a_0 + a_1 x + a_2 x^2 + \cdots$

$$= a_0 + a_1 x + \frac{1}{6} a_1 x^3 + \frac{1}{40} a_1 x^5 + \frac{1}{336} a_1 x^7 + \cdots$$