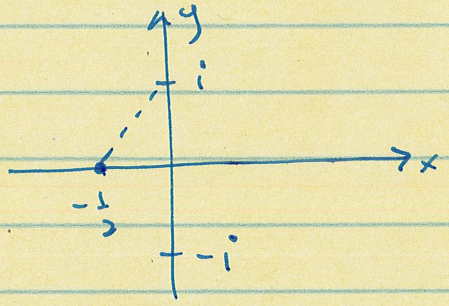
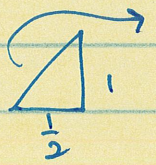
Quiz 2

1. $y'' + \frac{2x}{1+x^2} y' + \frac{4x^2}{1+x^2} y = 0$

Singular at $1+x^2=0 \rightarrow x = \pm i$ $\quad x_0 = -\frac{1}{2}$





$(\frac{1}{2})^2 + 1^2 = \frac{1}{4} + 1 = \frac{5}{4}$

$\rho = \sqrt{\frac{5}{4}} \left(\frac{\sqrt{5}}{2}\right)$

2. $y'' + \frac{3x}{2x(x-2)^2} y' + \frac{(x-2)}{2x(x-2)^2} y = 0$

Singular at $x=0, x=2$

$x=0$: $\lim_{x\to 0} x \cdot \frac{3x}{2x(x-2)^2} = 0$ $\quad \lim_{x\to 0} x^2 \cdot \frac{x-2}{2x(x-2)^2} = 0$

Regular

$x=2$: $\lim_{x\to 2} (x-2) \cdot \frac{3x}{2x(x-2)^2} \rightarrow = \infty$ Irregular

3. $2x^2y'' + 3xy' - y = 0 \qquad x>0$

$x^2y'' + \frac{3}{2}xy' - \frac{1}{2}y = 0$

$r(r-1) + \frac{3}{2}r - \frac{1}{2} = 0$

$2r(r-1) + 3r - 1 = 0$

$2r^2 + r - 1 = 0$

$$(2r - 1)(r + 1) = 0$$

$$r = \frac{1}{2}, \quad r = -1$$

$$y = c_1 |x|^{1/2} + c_2 |x|^{-1}$$