Quiz 4

1. $y''+4y = \sin t - u_{2\pi}(t)\cdot\sin(t-2\pi) \qquad y(0)=y'(0)=0$

$$(s^2+4)Y = \frac{1}{s^2+1} - e^{-2\pi s}\frac{1}{s^2+1}$$

$$Y = \frac{1}{(s^2+4)(s^2+1)} - e^{-2\pi s}\frac{1}{(s^2+4)(s^2+1)}$$

$$\frac{1}{(s^2+4)(s^2+1)} = \frac{As+B}{s^2+4} + \frac{Cs+D}{s^2+1}$$

$$1 = (As+B)(s^2+1) + (Cs+D)(s^2+4)$$
$$= As^3+As+Bs^2+B+Cs^3+4Cs+Ds^2+4D$$
$$1 = (A+C)s^3+(B+D)s^2+(A+4C)s+(B+4D)$$

$A+C=0$

$B+D=0$

$A+4C=0$

$B+4D=1 \qquad 3D=1 \qquad D=\frac{1}{3}, \quad B=-\frac{1}{3}$

$3C=0 \qquad C=0, \quad A=0$

$$\frac{1}{(s^2+4)(s^2+1)} = -\frac{1}{3}\frac{1}{s^2+4} + \frac{1}{3}\frac{1}{s^2+1}$$

$$y = -\frac{1}{6}\sin 2t + \frac{1}{3}\sin t - u_{2\pi}(t)\cdot\left[-\frac{1}{6}\sin 2(t-2\pi) + \frac{1}{3}\sin(t-2\pi)\right]$$

2. $y'' + 4y = \delta(t-\pi) - \delta(t-2\pi) \qquad y(0) = y'(0) = 0$

$$(s^2+4)Y = e^{-\pi s} - e^{-2\pi s}$$

$$Y = e^{-\pi s}\frac{1}{s^2+4} - e^{-2\pi s}\frac{1}{s^2+4}$$

$$y = u_\pi(t)\cdot\frac{1}{2}\sin 2(t-\pi) - u_{2\pi}(t)\cdot\frac{1}{2}\sin 2(t-2\pi)$$