

## Quiz 5

1.  $y'' + 4y = f(t) \quad y(0) = 3, \quad y'(0) = -1$

$$s^2 Y - sy(0) - y'(0) + 4Y = G(s)$$

$$(s^2 + 4)Y = 3s - 1 + G(s)$$

$$Y = \frac{3s}{s^2 + 4} - \frac{1}{s^2 + 4} + \frac{1}{s^2 + 4} G(s)$$

$$y(t) = 3 \cos 2t - \frac{1}{2} \sin 2t + \int_0^t \frac{1}{2} \sin 2(t-\tau) g(\tau) d\tau$$

2.  $ty'' + ty' + (t^2 - 0.25)y = 0$

$$x_1 = y$$

$$x_1' = x_2$$

$$x_2 = y'$$

$$x_2' = \frac{-tx_2 - (t^2 - 0.25)x_1}{t}$$

$$x_2' = -\frac{t^2 - 0.25}{t} x_1 - x_2$$

3.  $\begin{bmatrix} 3 & -1 \\ 4 & -2 \end{bmatrix} \quad \begin{vmatrix} 3-\lambda & -1 \\ 4 & -2-\lambda \end{vmatrix} = 0 \quad (3-\lambda)(-2-\lambda) + 4 = 0$

$$\lambda^2 - \lambda + 2 = 0$$

$$(\lambda - 2)(\lambda + 1) = 0$$

$\lambda = -1$

$$\begin{bmatrix} 4 & -1 & | & 0 \\ 4 & -1 & | & 0 \end{bmatrix}$$

$$\vec{v} = \begin{bmatrix} \frac{1}{4}r \\ r \end{bmatrix}$$

$\lambda = -1, \lambda = 2$

$$= \begin{bmatrix} 1 \\ 4 \end{bmatrix} \quad (r=1)$$

$\lambda = 2$

$$\begin{bmatrix} 1 & -1 & | & 0 \\ 4 & -4 & | & 0 \end{bmatrix}$$

$$\vec{v} = \begin{bmatrix} r \\ r \end{bmatrix}$$

$$= \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$(r=1)$