Quiz 7

1. $\vec{x}' = \begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix} \vec{x} + \begin{bmatrix} 2e^{-t} \\ 3t \end{bmatrix}$

$\Psi = \begin{bmatrix} e^{-3t} & e^{-t} \\ -e^{-3t} & e^{-t} \end{bmatrix}$ gen. solution : $\vec{x} = \Psi \vec{u}$

where $\Psi \vec{u}' = \vec{g}$

$$\left[\begin{array}{cc|c} e^{-3t} & e^{-t} & 2e^{-t} \\ -e^{-3t} & e^{-t} & 3t \end{array}\right]$$

$$\left[\begin{array}{cc|c} e^{-3t} & e^{-t} & 2e^{-t} \\ 0 & 2e^{-t} & 3t+2e^{-t} \end{array}\right] \quad \left[\begin{array}{cc|c} 1 & e^{2t} & 2e^{2t} \\ 0 & 1 & \frac{3}{2}te^{t}+1 \end{array}\right]$$

$u_2' = \frac{3}{2}te^{t}+1 \qquad u_2 = \int \frac{3}{2}te^{t}dt + t + C_2$

$u = t \quad dv = e^{t}dt$

$du = dt \quad v = e^{t}$

$= \frac{3}{2}te^{t} - \frac{3}{2}\int e^{t}dt + t + C_2$

$= \frac{3}{2}te^{t} - \frac{3}{2}e^{t} + t + C_2$

$u_1' = -e^{2t}u_2' + 2e^{2t}$

$= -e^{2t}\left(\frac{3}{2}te^{t}+1\right) = -\frac{3}{2}te^{3t} - e^{2t} + 2e^{2t}$

$u_1 = \frac{1}{2}e^{2t} - \frac{3}{2}\int te^{3t}dt \qquad u = t \quad dv = e^{3t}dt$

$du = dt \quad v = \frac{1}{3}e^{3t}$

$= \frac{1}{2}e^{2t} - \frac{3}{2}\left(\frac{1}{3}te^{3t} - \frac{1}{3}\int e^{3t}dt\right)$

$= \frac{1}{2}e^{2t} - \frac{1}{2}te^{3t} - \frac{1}{6}e^{3t} + C_1$

$\vec{x} = \Psi \vec{u}$. particular solution if $C_1 = C_2 = 0$

2. $y' = 2y - 3t \quad y(0) = 1 \qquad h = 0.5$

$$y(0.5) = y(0) + (2 \cdot y(0) - 3 \cdot 0)(0.5)$$
$$= 1 + 1 = 2$$

~~$y(0.5)$~~

$$y(1) = y(0.5) + (2 \cdot y(0.5) - 3 \cdot 0.5)(0.5)$$
$$= 2 + (4 - 1.5)(0.5)$$
$$= 2 + (2.5)(0.5) = 2 + 1.25 = 3.25$$