1.1 Some Basic Mathematical Models : Direction Field

differential equation : an equation that contains derivatives

for example, $\dfrac{dy}{dx} = \cos x$

$y' = x^2 + 2x - 5$

$\Big\}$ calculus

more complicated examples : $\dfrac{d^2 r}{dt^2} = -\dfrac{GM}{r^2}$ Newton's Law of Gravitation

G : universal gravitation constant

M : mass of attracting body (Earth)

diff. eqs. are often used to model dynamical situations (changing)

population change : P(t) is population

$$\frac{dP}{dt} = kP$$

k : constant of proportionality ($k > 0$)

eq says : rate of change of pop. is proportional to pop. size.

more realistic model: logistic growth

$P(t)$: pop.

L : limit of population (environmental)

$$\frac{dP}{dt} = \underbrace{K}_{\text{is proportional to}} (L - P)$$

rate of change

if $P < L$ (still room to grow)

then $(k > 0)$ $\frac{dP}{dt} > 0$ grow

when P is close to L, $\frac{dP}{dt}$ is small

when $P = L$, $\frac{dP}{dt} = 0$ (no change)

$P > L \rightarrow \frac{dP}{dt} < 0$ (decline)

goal: solve the diff. eq.

Solution is a function that satisfies the diff. eq.

$\frac{dy}{dx} = \cos x$ has solution $y = \sin x + C$

we will learn techniques to solve more complicated ones like $\frac{dP}{dt} = k(L-P)$

let's see how we can qualitatively understand the solution w/o solving the diff. eq.

for example, $y' = y$ we can't solve it like $\frac{dy}{dx} = \cos x$

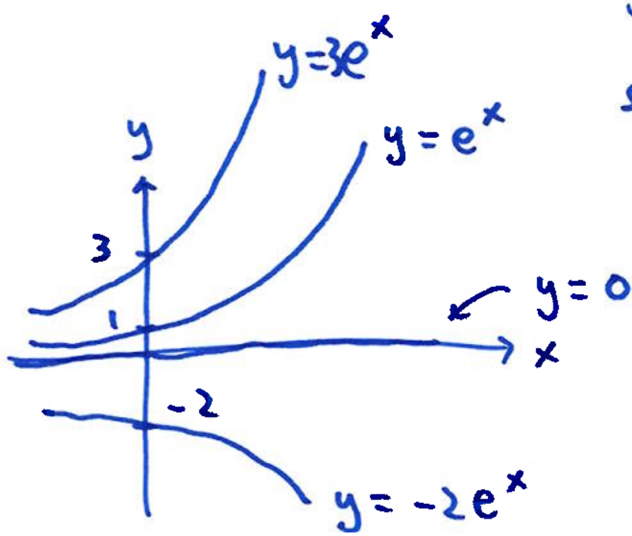
sometimes we can "guess" a solution

what does $y' = y$ say? we want a y ($y(x)$) such that it is its own derivative

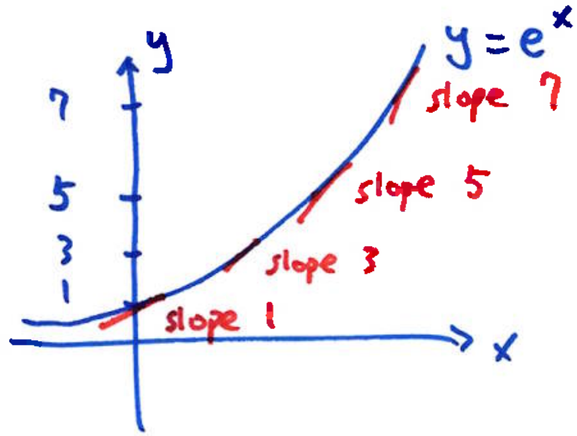
what is that function? $y = e^x$ so is $y = 2e^x$, $3e^x$, etc

so, $y = Ce^x$

graph:



$y' = y$ says the slope of tangent line on a solution curve is equal to the value of the function



notice even if we didn't know $y = Ce^x$, w/ enough slopes we can still "eye ball" the solution curves

→ Direction Field $\quad y' = y$

on the horizontal line

$y = 0, \quad y' = 0$

$y = 1, \quad y' = 1$

$y = 5, \quad y' = 5$