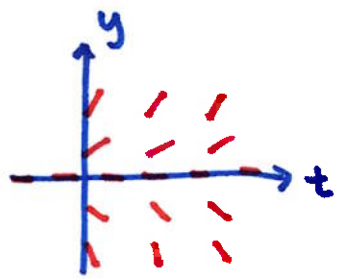
1.1 (Continued)

direction / slope field

from last time: $y' = y$ we graph the slope at different (t, y)

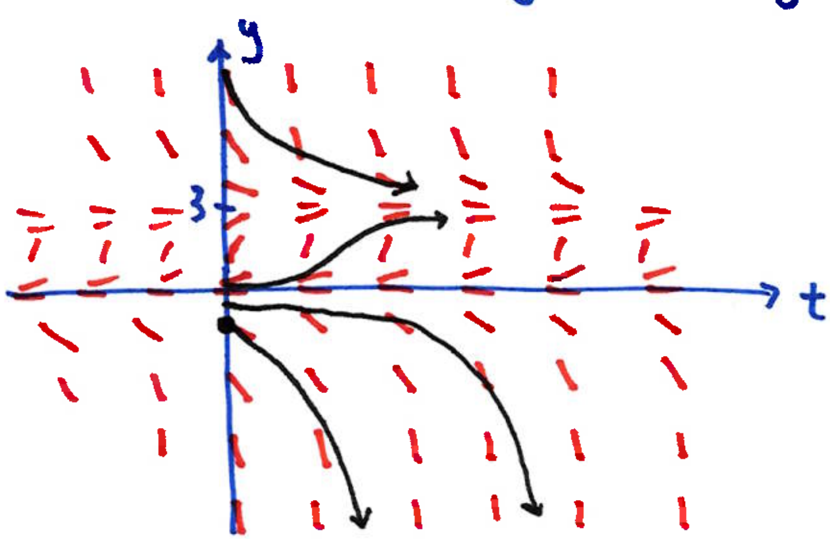


$y' = y$ does not depend on t

slopes don't change left / right

let's try $y' = y(3-y)$

first, notice $y' = 0$ at $y = 0$, $y = 3$ horizontal slopes



again, no t, so the same slopes on each horizontal line now we check the regions $y<0$, $0<y<3$, $y>3$

for $y<0$, $y' = y(3-y) < 0$

$- \quad +$

all negative slopes

near $y=0$, shallow negative slope

$y \to -\infty$, $y' \to -\infty$ as we go down, greater negative slopes

for $0<y<3$, $y' = y(3-y) > 0$

$+ \quad +$

starts nearly 0, increases as y increases, then decreases to 0 at $y=3$

for $y>3$, $y' = y(3-y) < 0$

$+ \quad -$

as $y \to \infty$, $y' \to -\infty$

steeper as we go up.

if $y(0) < 0$ (initial condition) $t \to \infty$, $y \to -\infty$

if $0 < y(0) < 3$ $t \to \infty$, $y \to 3$

if $y(0) > 3$, $t \to \infty$, $y \to 3$

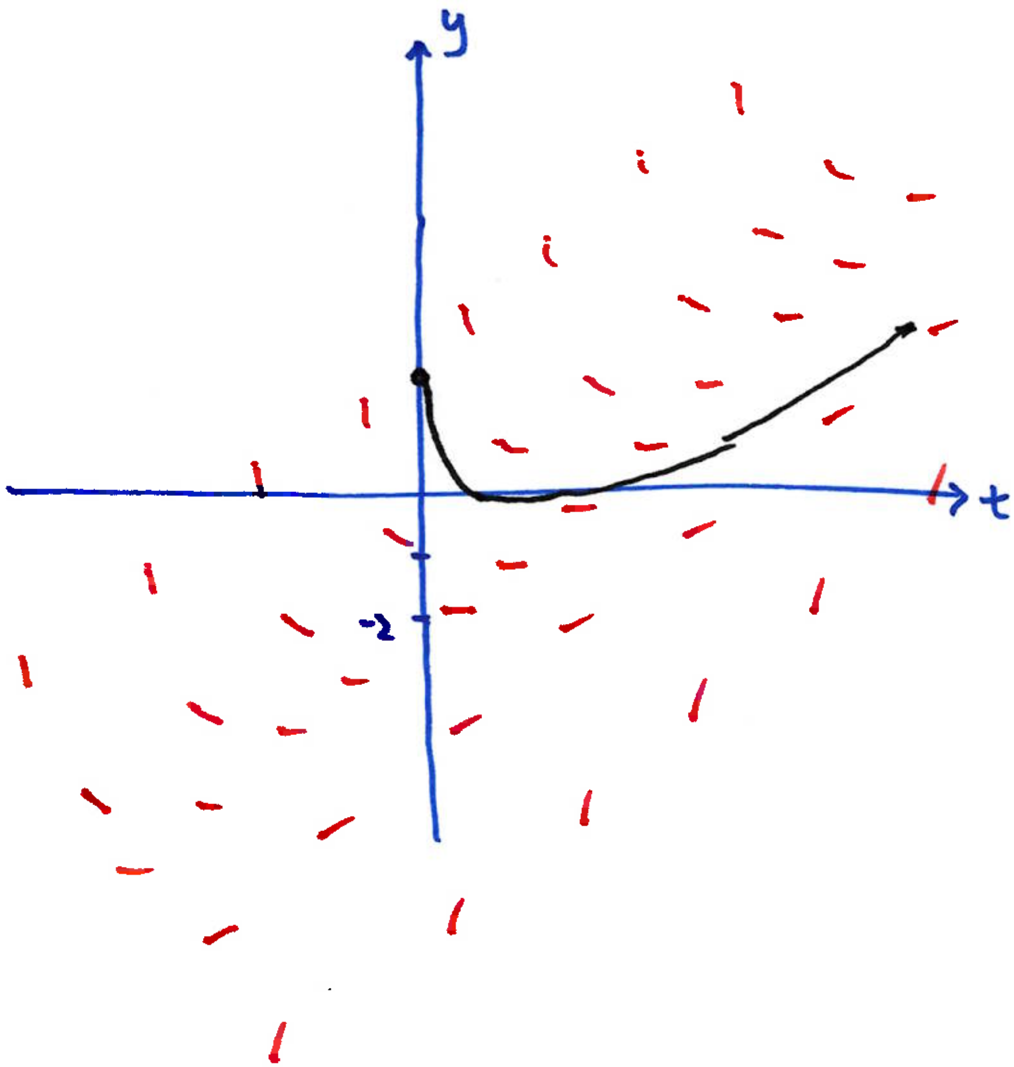
let's try $y' = -2 + t - y$

slopes depend on both t and y

efficient way to analyze: find out where $y' = 0$

$0 = -2 + t - y$

$y = t - 2$ $\rightarrow$ on this line slopes (y') are zero



$y' = (t-2) - y$

above $t-2$, $y > t-2$

$y' < 0$

more above, more negative

below $t-2$, $y < t-2$

$y' > 0$

more positive below

1.2 Solutions of Some Differential Eqs.

$$\frac{dy}{dt} = ay - b \qquad a, b \text{ constants}$$

shows up in free fall :

constant

air resistance = γv ← velocity

mass

gravity = mg

← gravity acceleration

Newton's 2nd Law : $F = ma$

v: ~~deci~~ velocity then $a = \frac{dv}{dt}$

$$mg - \gamma v = m\frac{dv}{dt}$$

rewrite : $\frac{dv}{dt} = g - \frac{\gamma}{m} v$

let's solve one : $\frac{dy}{dt} = -3y + 10$

can't solve by integrating both sides ~~direction~~ directly

(we don't know y)

but we can do this :

$$\frac{dy}{dt} = -3\left(y - \frac{10}{3}\right)$$

divide by $y - \frac{10}{3}$ $\left(y \neq \frac{10}{3}\right)$

$$\frac{1}{y - \frac{10}{3}} \frac{dy}{dt} = -3$$

let $u = y - \frac{10}{3}$ then $\frac{du}{dt} = \frac{dy}{dt}$

$$\frac{1}{u} \frac{du}{dt} = -3$$

integrate both sides with respect to t

$$\int \frac{1}{u}\frac{du}{dt}\,dt = \int -3\,dt$$

$$\int \frac{1}{u}\,du = \int -3\,dt$$

$$\ln|u| = -3t + c$$

$$\ln\left|y - \frac{10}{3}\right| = -3t + c$$

$$\left|y - \frac{10}{3}\right| = e^{-3t+c} = e^{-3t}\cdot e^{c}$$

$$y - \frac{10}{3} = \underbrace{\pm e^{c}}_{\text{call it } C}\cdot e^{-3t} = Ce^{-3t}$$

$$\boxed{y = \frac{10}{3} + Ce^{-3t}}$$

general solution

it is a collection of infinitely-many solutions (one for each value of C)

to find C, we need one point on the curve

typically, $y(0) = y_0$ (initial condition)

this makes the problem an Initial-Value Problem (IVP)

$$y = \frac{10}{3} + Ce^{-3t}$$

$y(0) = y_0$ sub in $y = y_0$, $t = 0$

$$y_0 = \frac{10}{3} + C \quad \text{so,} \quad C = y_0 - \frac{10}{3}$$

then $\boxed{y(t) = \frac{10}{3} + \left(y_0 - \frac{10}{3}\right)e^{-3t}}$ particular solution