Some Basic Mathematical Models and Direction Field

differential equation : any equation containing derivative(s)

you have seen some : for example, $\frac{dy}{dx} = e^x$ or $y' = x^2 + 2x - 5$ } from calculus

of course, they get more complicated :

$$y'' + 3y' + y = 2$$ models a mass-spring-damper system with constant external force

we use diff. eqs to model situations we want to study

$$\frac{dy}{dt} = k\,y$$

constant of proportionality ("is proportional to")

rate of change of y

y itself

exponential growth ($k > 0$)

" decay ($k < 0$)

or $$\frac{dy}{dt} = k(L - y)$$

logistic growth

L : carrying capacity (limit)

goal : solve the diff. eq.

↳ find a function that satisfies the diff. eq.

for example, $$\frac{dy}{dx} = \cos x$$

find $y(x)$ that satisfies the diff. eq.

$$y(x) = \sin(x) + C$$

because $\frac{d}{dx}(\sin x + C) = \cos x$

but $y'' + 3y' + 2y = 5$ is harder to solve (we'll learn how in this course)

we will learn many techniques

but sometimes we can "guess" a solution

for example, $y' = y$ we cannot simply integrate both sides

$$\int y'\,dx = \int y\,dx$$

$$y = \underbrace{\int y\,dx}_{?}$$

$y' = y \longrightarrow$ says to find y such that it is its own derivative

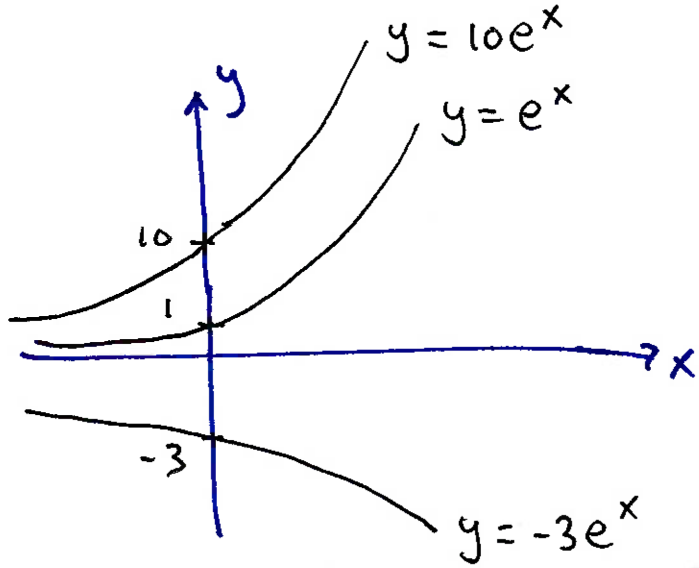
$y = e^x$

so is $y = 2e^x$ $(y' = 2e^x)$

so is $y = 3e^x$, $10e^x$, πe^x, etc

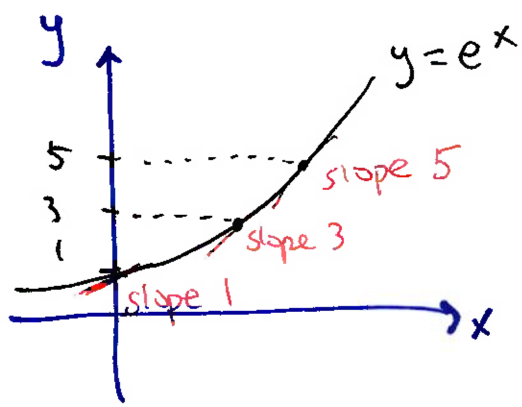
$$\boxed{y = Ce^x}$$ C : constant (C can be zero)

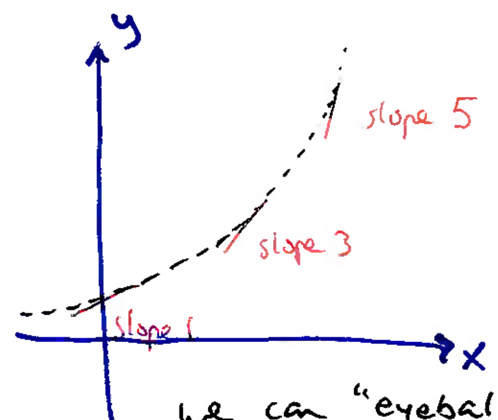
graph:



solution is often a <u>family of curves</u>

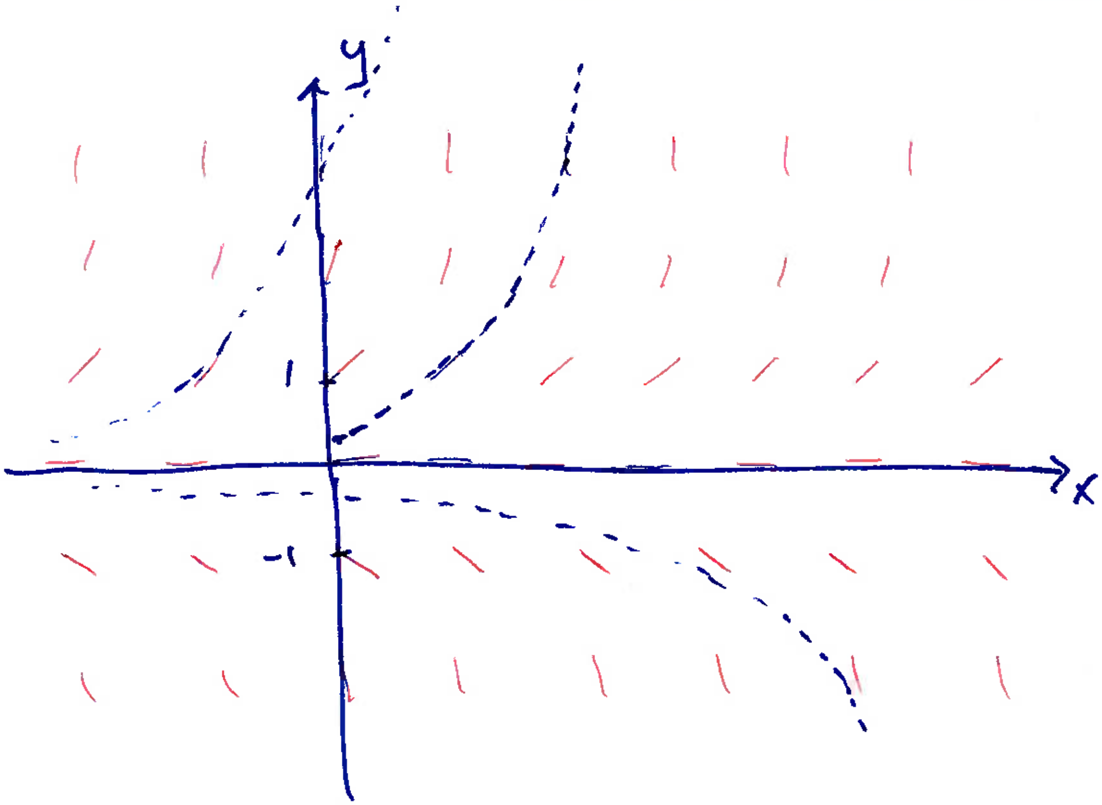
$y' = y$ → says that on the curve $y(x)$ the slope (y') is always equal to its y-value





we can "eyeball" solution from slopes

let's construct a slope field (direction field) for $y' = y$



$y' = y$

if $y = 1$ (horizontal line)
slopes at every x is 1

slope 1

$y = 0$ slopes $= 0$

the idea: use the slopes to visualize solutions qualitatively