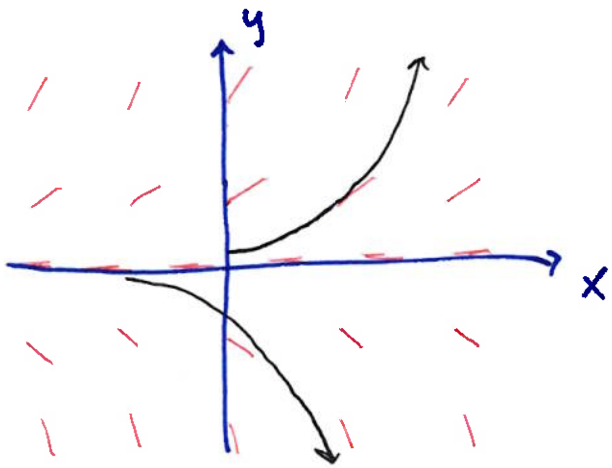
Direction Field / Slope Field (continued)

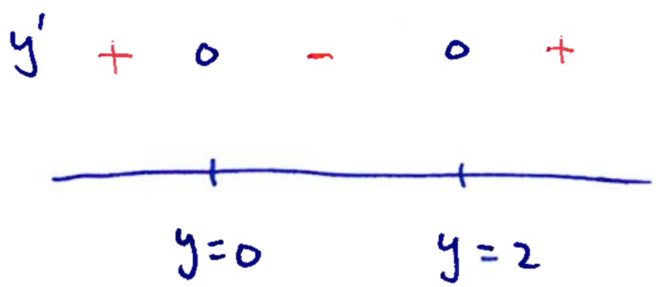
last time: $\frac{dy}{dx} = y$



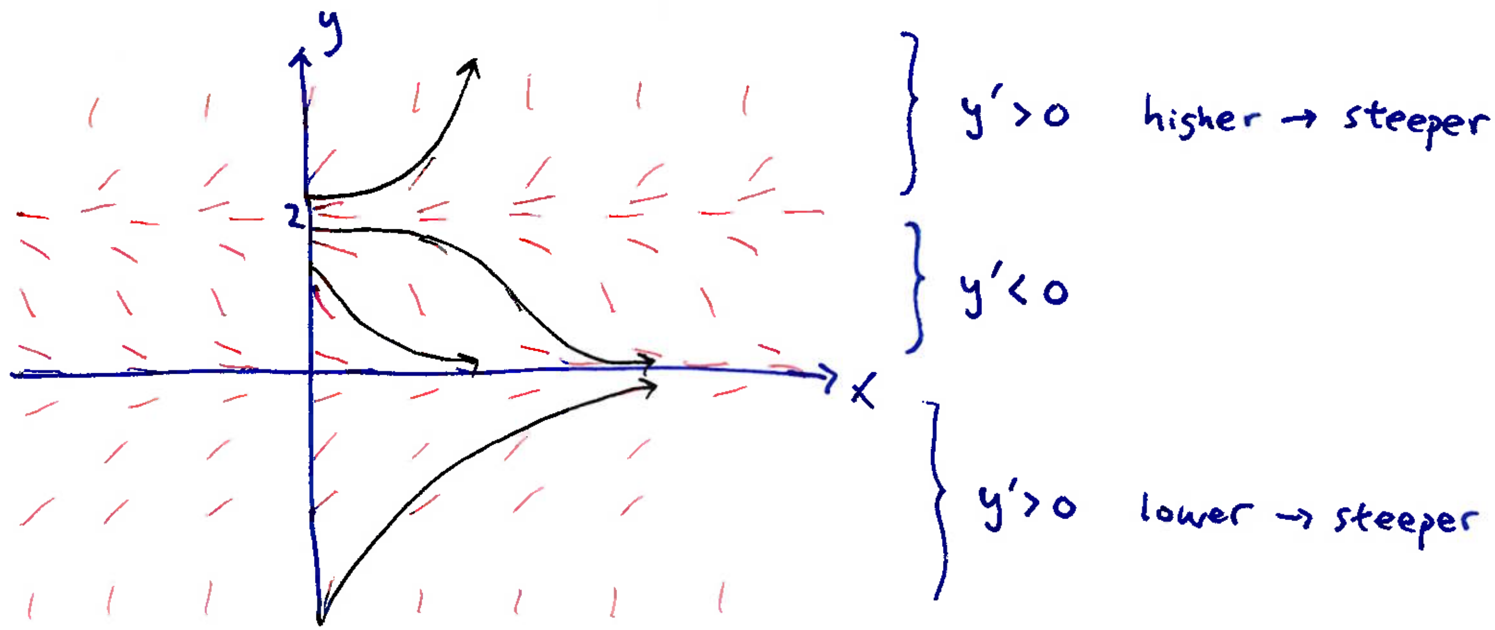
$y' = y$ (height)

let's look at another one: $y' = y(y-2)$

first, notice $y' = 0$ (slope $= 0$) at $y = 0$, $y = 2$



$y' = 0 \rightarrow$ equilibrium solutions



qualitatively, if the initial condition $y(0) > 2$

then as $x \to \infty$, $y \to \infty$

$0 < y(0) < 2$, then $x \to \infty$, $y \to 0$

$y(0) < 0$, then $x \to \infty$, $y \to 0$

we don't have the solution $y(x)$ but we understand its long term behavior let's look at $y' = -1 + t + y$ (t and x are common independent variables)

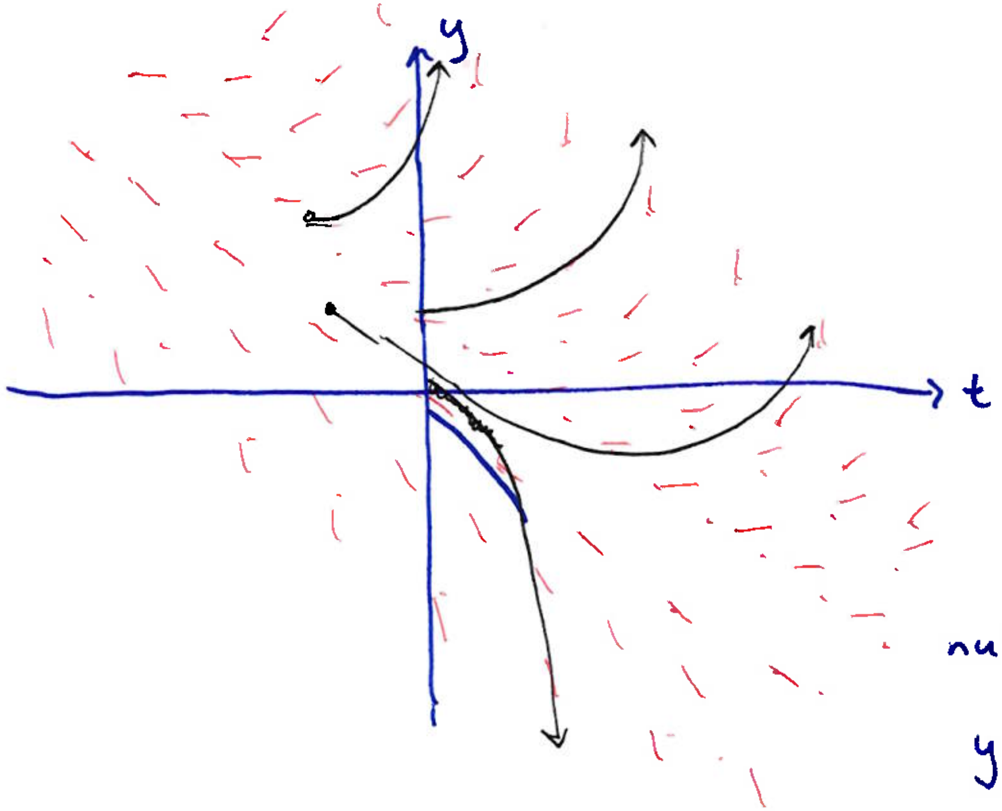
(if y' does not depend on the independent variable, then the diff. eq. is said to be <u>autonomous</u>)

if independent variable is present, it's usually best to find the curve ~~where~~ ~~y~~ on which y' is 0

$\longrightarrow$ <u>nullclines</u> (curve on which $y' = 0$)

$$y' = -1 + t + y$$

$$0 = -1 + t + y \rightarrow y = 1 - t \quad \text{nullcline}$$



$y' = -1 + t + y$

$y' = y - (1-t)$

if $y > 1-t$ then $y' > 0$

above nullcline, $y' > 0$

more above, the steeper

nullcline $y = 1-t$ $(y' = 0)$

$y' = y - (1-t)$

if $y < 1-t$ (below nullcline)

then $y' < 0$

more below, steeper slopes

this gives us some qualitative understandings

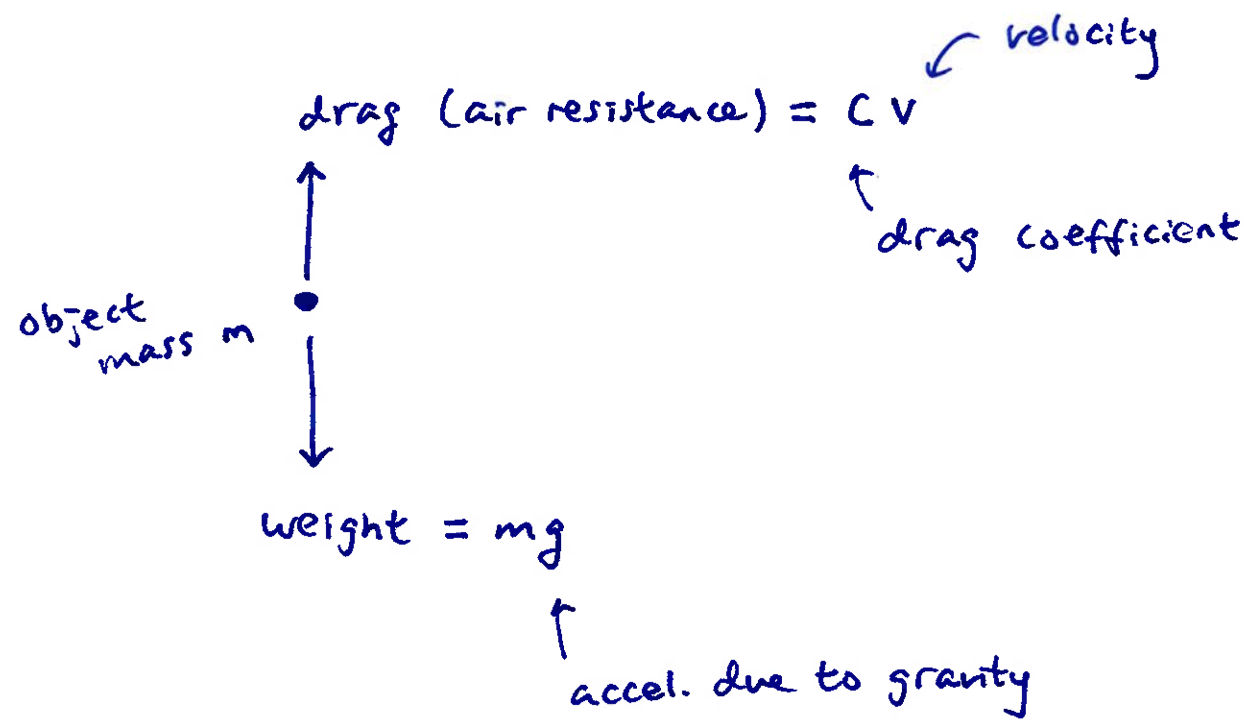
w/o solving the equation <u>analytically</u>

one equation we can already solve w/ calculus is

$$\frac{dy}{dt} = ay + b \qquad a, b \text{ constants}$$

simple but can be used to model many physical situations

for example, free fall



if y is the height, then $v = \frac{dy}{dt}$

Newton's 2nd Law: $F = m\underset{\text{accel.}}{a} = m\frac{dv}{dt}$

$$m\frac{dv}{dt} = mg - cv \longrightarrow \boxed{\frac{dv}{dt} = g - \frac{c}{m}v} \qquad \frac{dy}{dt} = ay + b$$

another application: Newton's Law of Cooling (plus more)

let's solve $\quad \frac{dy}{dt} = -2y + 5 \quad$ can't just integrate with respect to t directly

manipulate it a bit

$$\frac{dy}{dt} = -2\left(y - \frac{5}{2}\right)$$

divide by $y - \frac{5}{2}$ $\quad (y \neq \frac{5}{2})$

$$\frac{1}{y - \frac{5}{2}} \frac{dy}{dt} = -2$$

let $u = y - \frac{5}{2}$ then $\frac{du}{dt} = \frac{dy}{dt}$

the equation above becomes

$$\frac{1}{u} \frac{du}{dt} = -2$$

$$\int \frac{1}{u} \underbrace{\frac{du}{dt} \, dt}_{du} = \int -2 \, dt$$

$$\int \frac{1}{u} \, du = \int -2 \, dt$$

$$\ln |u| = -2t + C$$

$$|u| = e^{-2t+c} = e^{-2t} \cdot e^{c}$$

$$u = \underbrace{\pm e^{c}}_{\substack{\text{constant} \\ \text{call it } K}} \cdot e^{-2t} = Ke^{-2t}$$

$$u = y - \frac{5}{2}$$

so, $\boxed{y(t) = \frac{5}{2} - Ke^{-2t}}$