Solution to $y' = ay + b$ (continued)

last time: $\frac{dy}{dt} = -2y + 5$

Solution: $y(t) = \frac{5}{2} + Ce^{-2t}$ "general solution"

→ infinitely-many curves, depending C

to find C, normally we need an <u>initial condition</u> $y(0) = y_0$

→ this is now an initial-value problem (IVP)

for example, suppose $y(0) = 1$ $y' = -2y + 5$

$$y(t) = \frac{5}{2} + Ce^{-2t}$$

$$y(0) = 1 = \frac{5}{2} + Ce^{0} = \frac{5}{2} + C \qquad C = -\frac{3}{2}$$

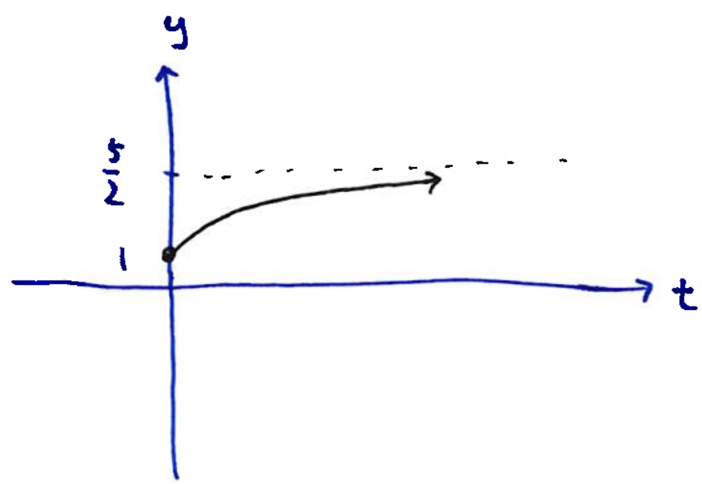
so, $y(t) = \frac{5}{2} - \frac{3}{2}e^{-2t}$ "particular solution" (one out of the general) let's interpret $\frac{dy}{dt} = -2y + 5$ in the context of free fall

$$\frac{dv}{dt} = g - \frac{c}{m} v$$

$\frac{dy}{dt} = -2y + 5$ is equivalent to $g = 5$

$\frac{c}{m} = 2$ (resistance due to air is twice the mass)

$y(t) = \frac{5}{2} - \frac{3}{2} e^{-2t}$ $\quad y(0) = 1$



as $t \to \infty$, $e^{-2t} \to 0$

so $y \to \frac{5}{2}$

in this case, we started w/ initial velocity of 1

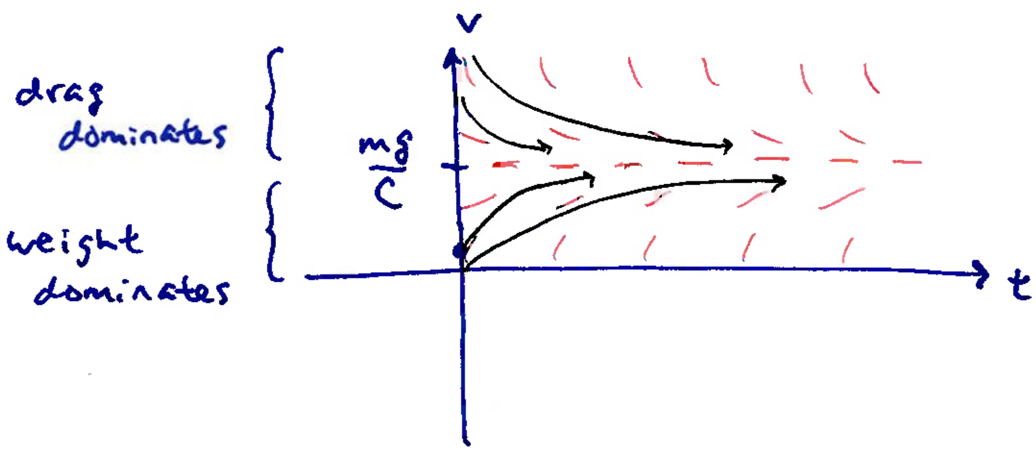
then as time goes on, we pick up more velocity

and eventually approach $\frac{5}{2}$ but never more

→ terminal velocity

$\frac{dv}{dt} = g - \frac{c}{m}v$ constant battle between g (gravity)

and $\frac{c}{m}v$ (drag)



$v' = 0 \rightarrow v = \frac{mg}{c}$

another interpretation of $\frac{dy}{dt} = ay + b$

in terms of population

rate is proportional to a times size plus a constant addition $(b > 0)$ at all times

Classification of Diff. Eqs.

two main types: ordinary diff. eq. (ODE)

- only contain ordinary derivatives
- solution is single-variable

partial diff. eq. (PDE)

- can contain partial derivatives
- solution is multivariate

in MA 36600, we study ODE (in MA 30300 there is PDE)

some example ODEs: $\frac{dv}{dt} = g - \frac{c}{m} v$ solution: $v(t)$

$\frac{dT}{dt} = K(T - T_{env})$ Newton's Law of Cooling

solution $T(t)$

example PDE : $a^2 \dfrac{\partial^2 y(x,t)}{\partial x^2} = \dfrac{\partial^2 y(x,t)}{\partial t^2}$ wave eq.

solution : $y(x,t)$

<u>order</u> of a diff. eq. : order of the highest derivative

$$\frac{dv}{dt} = g - \frac{c}{m} v \qquad \text{1st order}$$

$$my'' + cy' + ky = 0 \qquad \text{2nd order}$$

wave eq. is 2nd order (PDE)

<u>linear vs. nonlinear</u> : • in linear eqs. the coefficients of the dependent variable and its derivatives (e.g. y, y', y'', etc) <u>do not</u> contain the dependent variable and its derivatives

for example, $x^2 y'' + (\cos x) y' + 5 y = e^x$

do these contain y, y', y'', etc ?

NO. So this eq. is **linear**

$y''' + (\sin x) y = e^{x^2} \tan x$ is linear

$y'' + (y')^2 + y = 0$

$y'' + (y') y' + y = 0$ is **nonlinear**

coeff. y' contains y'

$y'' + \sin y = 0$

this makes the eq. nonlinear

$y'' = -\sin y$ (like e^x in 1st example but it contains y)

<u>Solution</u> : a function that satisfies the diff. eq.

for example, $y = e^t$ is <u>a</u> solution to $y' = y$

(so is $2e^t$, $3e^t$, etc)

for example, $y'' = y$ $y = e^t$ and $y = e^{-t}$

are two possible solutions

$y'' = -y$ $y = \cos x$ is a solution

so is $y = \sin x$

(part of this course is on techniques to find the solutions systematically)

<u>Homogeneous vs. Nonhomogeneous</u> : if written in the form of $F(y, y', y'', y''', \text{etc}) = g(x)$ where $g(x) = 0$ then the eq. is <u>homogeneous</u>

$$\underbrace{y'' + 2y' - 3y}_{F(y,\, y',\, y'')} = \underbrace{\cos x}_{\text{is this } 0\text{ ?}}$$

if so, eq. is homogeneous

if not, it's nonhomogeneous

$y'' + 2y' - 3y = 0$ is homogeneous