23. skydiver weighs 180 lb from 5000 ft.

opens chute after 10 seconds of free fall.

drag: $0.75|v|$ when chute closed

$12|v|$ " " open

define down as positive (because falling downward so v is always positive)

$$m\frac{dv}{dt} = \begin{cases} mg - 0.75v & \text{closed (first 10 seconds)} \\ mg - 12v & \text{open (after first 10 s)} \end{cases}$$

weight $= 180 = mg$

$= m(32)$ $\qquad m = \frac{180}{32} = 5.625$

$v(0) = 0$

a) speed when chute opens

$$\frac{dv}{dt} = g - \frac{0.75}{m} v = 32 - \frac{2}{15} v$$ separable / linear

$\vdots$

$$V = 240 + Ce^{-\frac{2}{15}t}$$

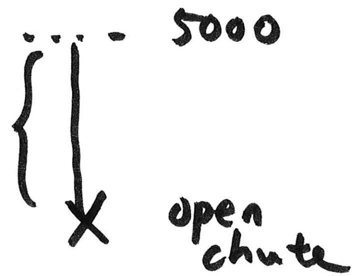
IC: $V(0) = 0$

$0 = 240 + C \qquad C = -240$

$$V(t) = 240 - 240e^{-\frac{2}{15}t} \qquad 0 \le t \le 10$$

$V(10) = 176.7$ ft/s

b). distance fallen before chute opens



y: distance fallen $\qquad y(0) = 0$

$$y(t) = \int v(t)\,dt = 240t + 1800e^{-\frac{2}{15}t} + C$$

$0 = 1800 + C \qquad C = -1800$

$$y(t) = 240t + 1800e^{-2/15 t} - 1800$$

$y(10) = 1074.47$ ft

c) terminal velocity after chute opens

$$\frac{dv}{dt} = g - \frac{12}{m} v$$

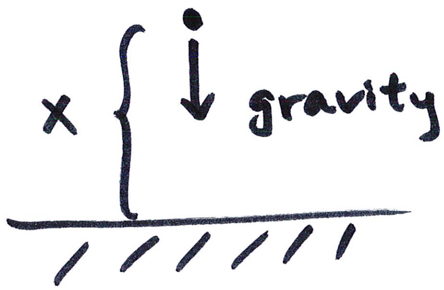
$\vdots$

$$v(t) = 15 + Ce^{-\frac{32}{15}t} \qquad \lim_{t \to \infty} v(t) = 15 \text{ ft/s}$$

29. $V_0 = \sqrt{2gR}$ R: radius of Earth

$\uparrow$ x pos. up

a).



surface : $x = 0$

$$\text{gravity} : -\frac{m\overset{\text{constant}}{\downarrow}{K}}{(R+x)^2}$$

for Earth, at $x = 0$

gravity is $-mg$

so $+\cancel{m}g = +\dfrac{\cancel{m}K}{R^2}$

$K = gR^2$

DE: $m\dfrac{dv}{dt} = -\dfrac{mgR^2}{(R+x)^2}$ two indep. variables

change to ~~t~~ x (see example 4 in textbook)

⋮

$$V = \sqrt{\frac{2gR^2}{R+x}}$$

b). time to reach $X = 240{,}000$ $R = 4000$

$$V = \sqrt{\frac{2gR^2}{R+X}} = \frac{dx}{dt}$$ find $X(t)$

$$\sqrt{R+X}\, dx = \sqrt{2gR^2}\, dt$$ separable.

$$\frac{2}{3}(R+X)^{3/2} = \sqrt{2gR^2}\, t + C$$ IC: $X(0) = 0$ (surface)

$$C = \frac{2}{3} R^{3/2}$$

240,000

$t \approx 51$ hours

2.4 Differences Between Linear and Nonlinear Eqs

linear: $y' + p(t)\,y = g(t) \qquad y(t_0) = y_0$

$$\mu(t) = e^{\int p(t)\,dt}$$

$$\frac{d}{dt}\left[\mu(t)\,y\right] = \mu(t)\,g(t)$$

solution: $\mu(t)\,y = \int \mu(t)\,g(t)\,dt + C$

$$y = \frac{1}{e^{\int p(t)dt}} \int e^{\int p(t)dt} g(t)\,dt + C$$

solution only exists if $\int p(t)\,dt$ and $\int e^{\int p(t)dt} g(t)\,dt$ exist $\rightarrow$ $\cancel{\int p(t)\,dt \text{ and } \int}$.

$p(t)$ and $g(t)$ are <u>both continuous</u>

Solution exists and is unique on any interval on which $p(t)$ and $g(t)$ are continuous, AND containing $y(t_0) = y_0$

<u>example</u>

$$y' + \frac{1}{t(t-3)} y = \sin t \qquad \boxed{y(1) = 2} \quad t = 1$$

$$p(t) = \frac{1}{t(t-3)}$$ continuous ~~on~~ where $t \neq 0, t \neq 3$

$g(t) = \sin t$ continuous for all t

$p(t)$ and $g(t)$ are both continuous on

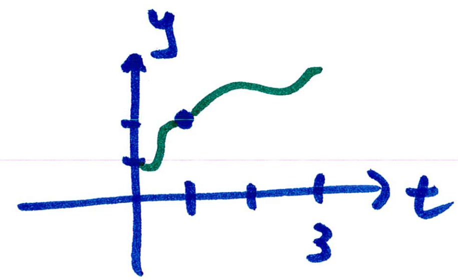
$(-\infty, 0)$, $\boxed{(0, 3),}$ $(3, \infty)$

$y(1) = 2$ is in here

so solution exists and is unique on $(0, 3)$

unique : one solution only

→ ~~d dey~~ solutions cannot intersect



for nonlinear eqs, $y' = f(t, y)$ $\quad y(t_0) = y_0$

Solution exists and is unique in a rectangle in ty-plane where f and $\frac{\partial f}{\partial y}$ are continuous and containing $\cancel{t = t_0}$ $y(t_0) = y_0$

example $y' = \dfrac{4t}{y-1}$

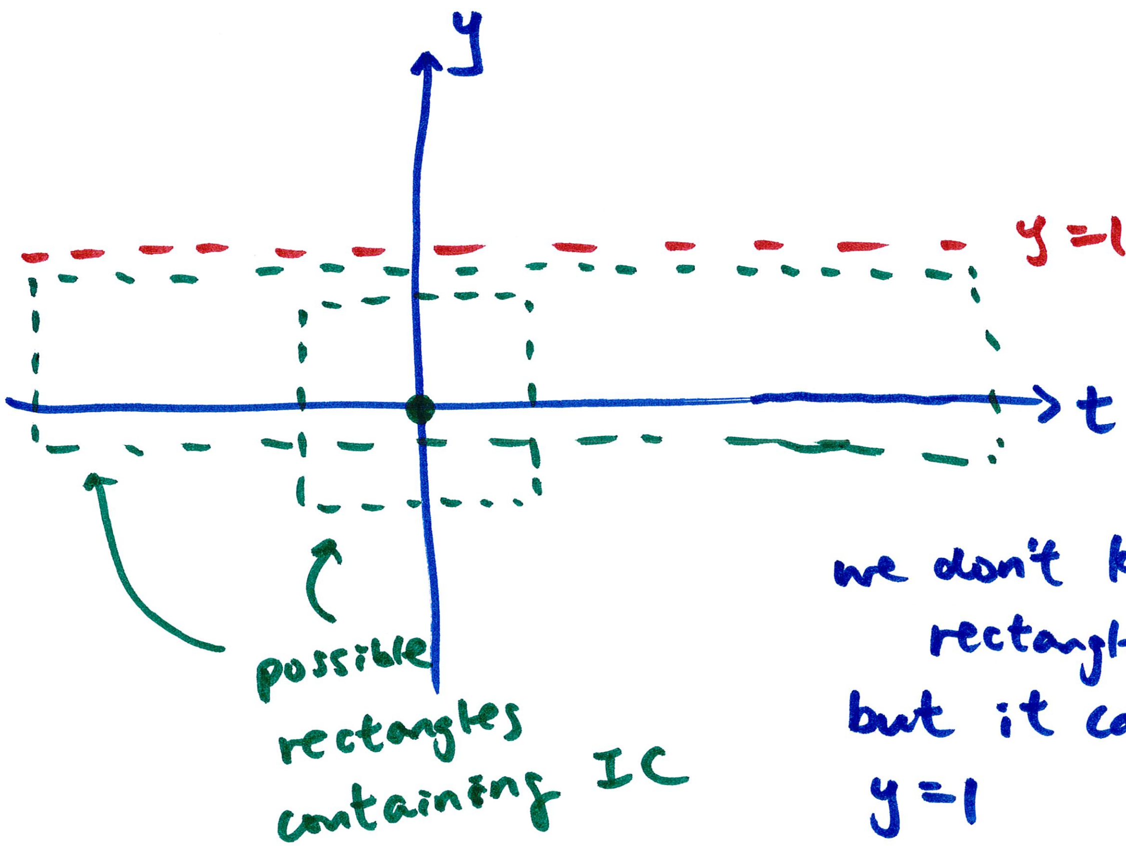
$$f(t,y) = \frac{4t}{y-1} \longrightarrow 4t(y-1)^{-1}$$

continuous $y \neq 1$

$$\frac{\partial f}{\partial y} = -\frac{4t}{(y-1)^2}$$

continuous $y \neq 1$

if $y(0) = 0$



we don't know how big rectangle is
but it can't grow past $y=1$

nonlinear eqs are usually very hard to solve

exceptions: some separable

Bernoulli equation

$\hookrightarrow$ $y' + p(t)y = g(t)y^n \quad n \neq 0, 1$

if $n=0$, $y' + p(t)y = g(t)$ linear

if $n=1$, $y' + p(t)y = g(t)y$

$$y' + \underbrace{[p(t) - g(t)]}_{\text{linear}} y = 0 \quad \text{linear.}$$

to solve Bernoulli, use subs: $V = y^{1-n}$

example $y' - y = -y^2$ $\qquad p(t) = -1 \quad q(t) = -1$

$n = 2$

$v = y^{1-2} = y^{-1}$

$v' = -y^{-2} y' \longrightarrow y' = -y^2 v'$

$-y^2 v' - y = -y^2$ $\qquad$ divide by $-y^2$

$v' + \frac{1}{y} = 1$ $\qquad v = y^{-1} = \frac{1}{y}$

$v' + v = 1$ $\qquad$ linear in v

$\vdots$

$v = 1 + Ce^{-t}$ $\qquad v = \frac{1}{y} \rightarrow y = \frac{1}{v}$

$$\boxed{y = \frac{1}{1 + Ce^{-t}}}$$