28. $y'' - y' - 2y = 0$

a). $y_1 = e^{-t}$, $y_2 = e^{2t}$

b). $y_3 = -2e^{2t}$ $\quad y_4 = y_1 + 2y_2 \quad y_5 = 2y_1 - 2y_3$

$= -2y_2$

all are linear combos of y_1 and y_2 (fundamental solutions) so all satisfy DE.

c). check Wronskian: $W \neq 0$, then they are fundamental solutions

3.3 Complex Roots of Characteristic Eq.

$y'' + ay' + by = 0$

char. eq. $\quad r^2 + ar^1 + b = 0 \quad \Rightarrow \quad r = r_1, r_2$

$y_1 = e^{r_1 t} \qquad y_2 = e^{r_2 t}$

if roots are complex : $\quad r = \lambda \pm i\mu$

<u>both</u> complex or <u>both</u> real

complex exponential : $\quad e^{a+bi} = e^{a}(\cos b + i \sin b)$

$e^{a} e^{bi}$

$y'' - 2y' + 2y = 0 \qquad r^2 - 2r + 2 = 0$

$$r = \frac{2 \pm \sqrt{4 - 4(1)(2)}}{2} = \frac{2 \pm \sqrt{-4}}{2}$$

$$= \frac{2 \pm 2i}{2} = 1 \pm i$$

$r_1 = 1 + i \qquad r_2 = 1 - i$

$$y_1 = e^{r_1 t} = e^{(1+i)t} = e^t e^{it} = e^t(\cos t + i \sin t)$$

complex value y_1

$$y_2 = e^{r_2 t} = e^{(1-i)t} = e^t \underbrace{e^{-it}}_{e^{i(-t)}} = e^t(\cos(-t) + i \sin(-t))$$

$$= e^t(\cos t - i \sin t)$$

general solution: $y = c_1 y_1 + c_2 y_2$

$$y = c_1 e^t(\cos t + i \sin t) + c_2 e^t(\cos t - i \sin t)$$

$\downarrow$ real

$$= \underbrace{(c_1 + c_2)}_{K_1} e^t \cos t + \underbrace{i\,(c_1 - c_2)}_{K_2} e^t \sin t$$

real and depend on IC's

c_1 and c_2 are complex conjugate pairs

$$\boxed{y = K_1 e^t \cos t + K_2 e^t \sin t}$$

in general, if $r = \lambda \pm i\mu$

then general solution is

$$\boxed{y = c_1 e^{\lambda t} \cos \mu t + c_2 e^{\lambda t} \sin \mu t}$$

cd. c_1, c_2 are real, depend on ICs.

<u>example</u> $4y'' + 9y = 0$

$4r^2 + 9 = 0 \qquad r = \pm \frac{3}{2} i = 0 \pm \frac{3}{2} i$

$$\boxed{y = c_1 \cos \tfrac{3}{2} t + c_2 \sin \tfrac{3}{2} t}$$

<u>example</u> $y'' + y' + 1.25y = 0 \qquad y(0)=3, \; y'(0)=1$

$$r^2 + r + 1.25 = 0$$

$$r = \frac{-1 \pm \sqrt{1-4(1)(1.25)}}{2} = \frac{-1 \pm \sqrt{-4}}{2} = -\frac{1}{2} \pm i$$

$$y = c_1 e^{-\frac{1}{2}t}\cos t + c_2 e^{-\frac{1}{2}t}\sin t$$

$$y' = c_1\left(-e^{-\frac{1}{2}t}\sin t - \frac{1}{2}e^{-\frac{1}{2}t}\cos t\right)$$

$$+ c_2\left(e^{-\frac{1}{2}t}\cos t - \frac{1}{2}e^{-\frac{1}{2}t}\sin t\right)$$

$y(0)=3$

$\rightarrow 3 = c_1$

$y'(0)=1$

$\rightarrow 1 = 3\left(-\frac{1}{2}\right) + c_2(1) \qquad c_2 = \frac{5}{2}$

particular solution

$$\boxed{y = 3e^{-\frac{1}{2}t}\cos t + \frac{5}{2}e^{-\frac{1}{2}t}\sin t}$$

$\lim_{t\to\infty} y = 0$ because of $e^{-\frac{1}{2}t}$

sin, cos $\to$ oscillations

combine $\to$ decaying oscillations

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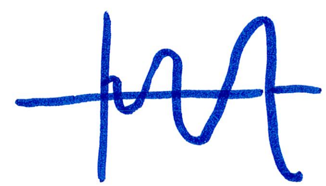
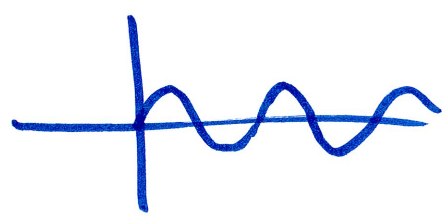
in general, $r = \lambda \pm i\mu$

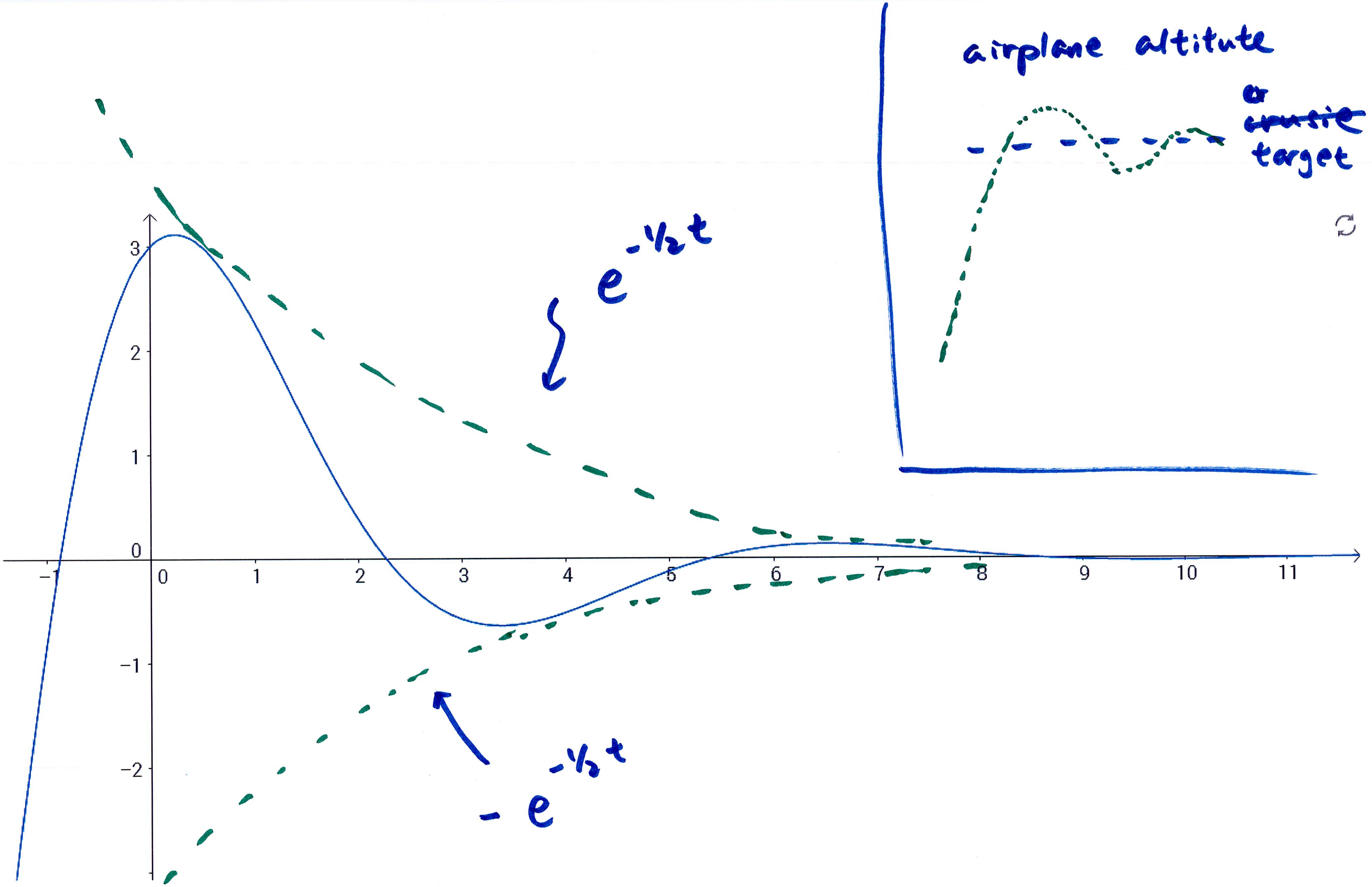
if $\lambda > 0$, growing ~~expo~~ oscillations



if $\lambda < 0$, decaying oscillations

if $\lambda = 0$, just oscillations





$e^{-\frac{1}{2}t}$
$-e^{-\frac{1}{2}t}$
airplane altitute
a
~~crusie~~
target
3
2
1
0
−1
−2
−1
0
1
2
3
4
5
6
7
8
9
10
11