1.3 # 4

$$\frac{dy}{dt} + ty^{2} = 0$$

not 1

nonlinear

$y \cdot y$

first order

1.2 # 8

$$\frac{dp}{dt} = rp$$

$$\frac{1}{p}\frac{dp}{dt} = r$$

$$\frac{d}{dt}\ln|p| = r$$

$$\ln P = rt + A$$

$$p(t) = e^{rt+A} = e^{rt} \cdot e^{A} = Ce^{rt}$$

C

$$P(0) = P_0$$

$$P_0 = C$$

$$p(t) = P_0 e^{rt}$$

$$2\cancel{P_0} = \cancel{P_0} e^{30r}$$

$$\ln 2 = 30r$$

$$\boxed{r = \frac{\ln 2}{30}}$$

≈ 0.0231

(2.31%)

Supp. A

$\cancel{t^2y'' - 4ty' + 4y = 0}$

$y = \underbrace{Be^{-2t}}$ $\qquad 2y' + 4y = 3e^{-2t}$

$y' = \underbrace{-2Be^{-2t}}$

$$2(-2Be^{-2t}) + 4(Be^{-2t}) = 3e^{-2t}$$

$$-4Be^{-2t} + 4Be^{-2t} = 3e^{-2t}$$

$$0 = 3e^{-2t} \quad \text{not true}$$

no choice of B can make $y = Be^{-2t}$ satisfy the DE $\rightarrow$ so $y = Be^{-2t}$ cannot be a solution of the DE.

2.1 Linear equations, Integrating Factors

solve eqs of the form $y' + p(t)\,y = g(t)$ first order linear DE

example: $y' + \frac{2}{t} y = \frac{4}{t}$

notice if we multiply both sides by t^2

product rule

$$t^2 y' + 2ty = 4t$$

$$\frac{d}{dt}\left(t^2 y\right) = 4t$$

integrate: $t^2 y = 2t^2 + C$

so $\boxed{y = 2 + Ct^{-2}}$ $t \neq 0$

t^2 is called the <u>integrating factor</u> : different for every DE so what is the <u>integrating factor</u> for arbitrary

$$y' + p(t)y = g(t) \; ? \;\longrightarrow\; \mu(t)$$

(lowercase Greek "mu")

multiply by $\mu(t)$

$$\boxed{\mu(t)y' + \mu(t)p(t)y} = \mu(t)g(t)$$

$$\parallel$$

want left side to be $\boxed{\frac{d}{dt}\left[\mu(t)y\right] = \mu(t)y' + \mu'(t)y}$

$$\cancel{\mu(t)y'} + \mu(t)p(t)y = \cancel{\mu(t)y'} + \mu'(t)y$$

$$\mu(t)p(t)\cancel{y} = \mu'(t)\cancel{y}$$

$$\hookrightarrow \quad \frac{d\mu(t)}{dt} = \mu(t)p(t)$$

$$\frac{1}{\mu(t)} \frac{d\mu(t)}{dt} = p(t)$$

$$\frac{d}{dt} \ln|\mu(t)| = p(t)$$

$$\ln|\mu(t)| = \int p(t)\,dt$$

$$\boxed{\mu(t) = e^{\int p(t)\,dt}}$$

Integrating Factor

<u>example</u>

$$y' - 2y = t^2 e^{2t}$$

$$y' + p(t)\,y = g(t)$$

"standard form"

here, $p(t) = -2$

C=0 in int. factor

so $\mu(t) = e^{\int -2\,dt} = e^{-2t+C} = e^{-2t}$

multiply DE by $\mu(t)$

$$\frac{1}{e^{2t}}$$

$$e^{-2t}y' - 2e^{-2t}y = e^{-2t}t^2e^{2t}$$

Left side is always $\frac{d}{dt}\left[\mu(t)y\right]$

$$\frac{d}{dt}\left(\underbrace{e^{-2t}}_{\mu(t)}y\right) = t^2$$

Integrate

$$e^{-2t}y = \frac{t^3}{3} + C$$

$$\boxed{y(t) = \frac{t^3}{3}e^{2t} + Ce^{2t}}$$

general solution ($C = ?$)

example:

C depends on initial condition $y(0) = 1$

$$1 = 0 + C \qquad \text{so} \qquad C = 1$$

$$y(t) = \frac{t^3}{3}e^{2t} + e^{2t}$$

particular solution (C is known)

what happens if $t \to \infty$? $y \to \infty$

example $y' - \frac{1}{2} y = 2\cos t$

$p(t)$

see slope field

depending on $y(0)$, $y \to \infty$ as $t \to \infty$ or

$y \to -\infty$ as $t \to \infty$ or

y is oscillatory as $t \to \infty$

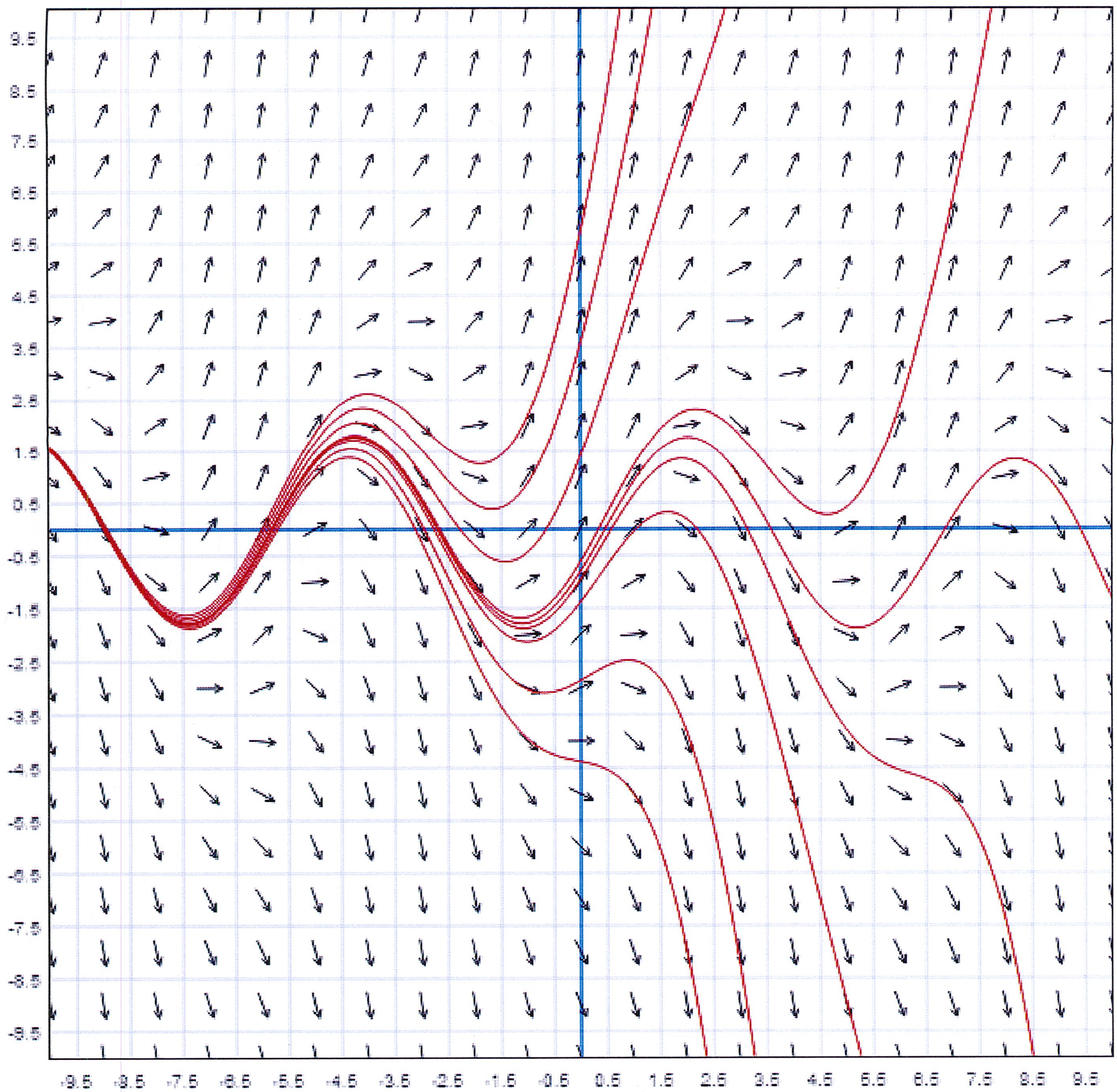
looks like the critical $y(0)$ is ≈ -0.75

let's find it.

$$\mu(t) = e^{\int -\frac{1}{2}\,dt} = e^{-\frac{1}{2}t}$$

$$e^{-\frac{1}{2}t} y' - \frac{1}{2} e^{-\frac{1}{2}t} y = 2e^{-\frac{1}{2}t} \cos t$$

$$\frac{d}{dt}\left(e^{-\frac{1}{2}t} y\right) = 2e^{-\frac{1}{2}t} \cos t$$



$$y' - \frac{1}{2}y = 2\cos t$$

LIATE

$$e^{-\frac{1}{2}t} y = \int 2e^{-\frac{1}{2}t} \cos t \, dt$$

integration by parts

$u = \cos t \qquad dv = 2e^{-\frac{1}{2}t} dt$

$du = -\sin t \, dt \qquad v = -4e^{-\frac{1}{2}t}$

$uv - \int v \, du$

$$e^{-\frac{1}{2}t} y = -4e^{-\frac{1}{2}t} \cos t + \underbrace{\int 4e^{-\frac{1}{2}t} \sin t \, dt}_{\text{by parts again}}$$

$\vdots$

$$= -\frac{4}{5} e^{-\frac{1}{2}t} (\cos t - 2\sin t) + C$$

$$y = -\frac{4}{5}(\cos t - 2\sin t) + Ce^{\frac{1}{2}t}$$

assume $y(0) = a$

$a = -\frac{4}{5} + C$ so $C = a + \frac{4}{5}$

$$y = \underbrace{-\frac{4}{5}(\cos t - 2\sin t)}_{\text{oscillation}} + \underbrace{\left(a+\frac{4}{5}\right)e^{\frac{1}{2}t}}_{\text{goes to } \infty \text{ if } a > -\frac{4}{5}}$$

goes to $-\infty$ if

$a < -\frac{4}{5}$

$a = -\frac{4}{5}$ to have

just oscillations