32. $2y' + ty = 2$

eq 47: $y = e^{-t^2/4} \int_0^t e^{s^2/4}\,ds + ce^{-t^2/4}$

$\lim_{t\to\infty} y = ?$

$$y = \frac{\int_0^t e^{s^2/4}\,ds}{e^{t^2/4}} \quad \begin{matrix} \to f(t) \\ \\ \to g(t) \end{matrix}$$

$\int_0^t s^2\,ds$

$= \frac{1}{3}s^3 \Big|_0^t$

$= \frac{1}{3}t^3$

$\int_0^t e^{s^2/4}$

$$\lim_{t\to\infty} y = \frac{\infty}{\infty}$$

use l'Hospital's Rule

deriv. of top & bottom

$$= \lim_{t\to\infty} \frac{e^{\cancel{t^2/4}}}{\frac{t}{2}e^{\cancel{t^2/4}}} = 0$$

24. $ty' + (t+1)y = 2te^{-t}$ $\quad y(1) = a, \ t > 0$

$\vdots$

$y = te^{-t} + ce^{-t}/t$

$a = e^{-1} + ce^{-1} = e^{-1}(1+c)$

$ae = 1 + c$ $\quad$ so $\quad c = ae - 1$

$$y = \underbrace{\frac{t}{e^t}}_{\substack{\to 0 \\ \text{as } t \to 0}} + \underbrace{(ae-1)}_{\text{sign?}} \underbrace{\frac{1}{te^t}}_{\substack{\to \infty \\ \text{as } t \to 0}}$$

$t \to 0$ $\quad y \to \infty$

$y \to -\infty$

$y \to 0$

$\hookrightarrow a = ?$

want $y \to 0$, so choose a so that $ae - 1 = 0$

$a = \frac{1}{e}$

2.2 Separable Equations

Hw #28 is optional

look at $y' = \dfrac{x^2}{y(1+x^3)}$

y: dependent variable

x: indep. variable

$$\frac{dy}{dx} = \frac{x^2}{y(1+x^3)}$$

multiply $y\,dx$ so each variable is on its own side

$$y\,dy = \frac{x^2}{1+x^3}\,dx$$

integrate

$$\int y\,dy = \int \frac{x^2}{1+x^3}\,dx$$

$u = 1+x^3$

$du = 3x^2\,dx$

$dx = \dfrac{1}{3x^2}\,du$

$$\frac{1}{2}y^2 = \int \frac{\cancel{x^2}}{u}\frac{1}{3\cancel{x^2}}\,du = \int \frac{1}{3u}\,du$$

$$= \frac{1}{3}\ln|u| + C$$

put $+C$ w/ x

$$\boxed{\frac{1}{2}y^2 = \frac{1}{3}\ln|1+x^3| + C}$$

implicit form of solution

often written as

multiply by 6 to eliminate fractions

$$3y^2 = 2\ln|1+x^3| + C$$

6 times previous C

$$\boxed{3y^2 - 2\ln|1+x^3| = C}$$

or $\quad y^2 = \frac{2}{3}\ln|1+x^3| + C$

$$y = \left(\frac{2}{3}\ln|1+x^3| + C\right)^{1/2}$$

explicit form

example

$$y' = \frac{1-2x}{y} \qquad y(1) = -2$$

$$\frac{dy}{dx} = \frac{1-2x}{y}$$

separate by multiplication/div
NEVER by add/subtraction

$$y\,dy = (1-2x)\,dx$$

$$\int y\,dy = \int (1-2x)\,dx$$

$$\frac{1}{2}y^2 = x - x^2 + C$$

use init. condition (IC)
$y(1) = -2$

$$\frac{1}{2}(-2)^2 = 1 - (1)^2 + C$$

so $C = 2$

$$\frac{1}{2}y^2 = x - x^2 + 2$$

$$y^2 = 2x - 2x^2 + 4$$

$$y = \pm\sqrt{2x-2x^2+4}$$

can have only one of these as a solution,

use IC: $y(1) = -2$, so here, choose negative

$$\boxed{y = -\sqrt{2x-2x^2+4}}$$

on what interval* is this solution valid?

can't take square root of neg. #

so $2x - 2x^2 + 4 \geq 0$

? why / why not?

NO, because $y = 0$ breaks the DE

$$y' = \frac{1-2x}{y}$$

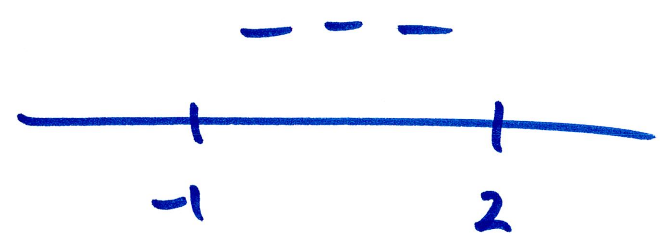
even though $y = -\sqrt{2x-2x^2+4}$ is still defined

$$x - x^2 + 2 > 0$$

$$-(x^2 - x - 2) > 0$$

$$(x^2 - x - 2) < 0$$

$$(x - 2)(x + 1) < 0$$

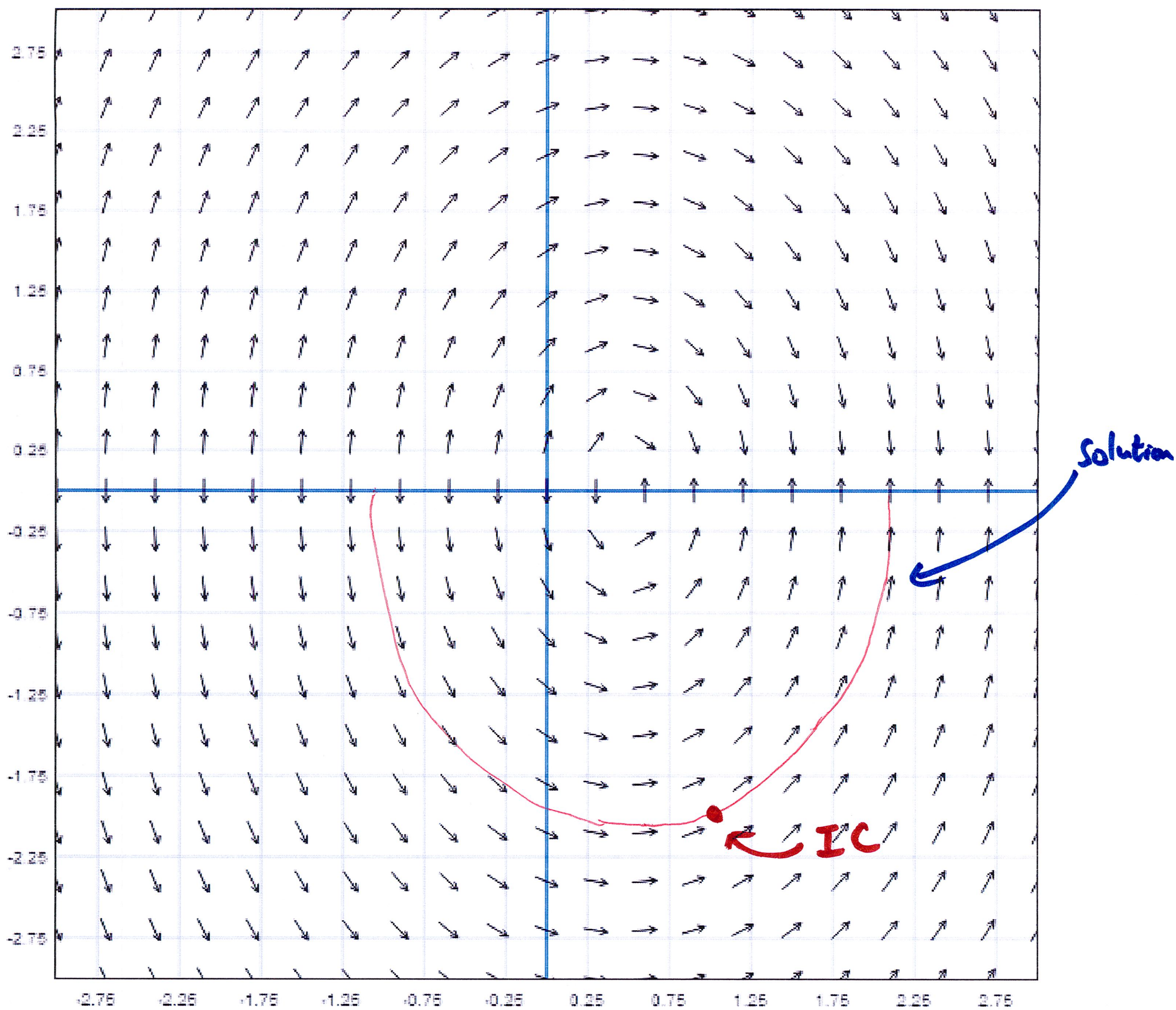


so interval is $-1 < x < 2$

at $x = -1$, and $x = 2$, $y' = \dfrac{1-2x}{y}$ is undefined

vertical tangent on $y(x)$

stay away from these


2.75
2.25
1.75
1.25
0.75
0.25
-0.25
-0.75
-1.25
-1.75
-2.25
-2.75
-2.75 -2.25 -1.75 -1.25 -0.75 -0.25 0.25 0.75 1.25 1.75 2.25 2.75
Solution
IC

$$y' = \frac{1-2x}{y}$$

Homogeneous equations

if $\frac{dy}{dx}$ can be expressed as a function of $\frac{y}{x}$

example: $$\frac{dy}{dx} = \frac{x^2+xy+y^2}{x^2}$$

$$= 1 + \frac{y}{x} + \left(\frac{y}{x}\right)^2$$

~~Homogeous~~

homogeneous eqs can be turned into separable by making a change of variable $y = x\,V(x)$

or $V(x) = \frac{y}{x}$

$$\frac{dy}{dx} = \frac{x^2 + xy + y^2}{x^2} = 1 + \frac{y}{x} + \left(\frac{y}{x}\right)^2$$

let $y = x\,v(x)$ so $\frac{dy}{dx} = \frac{d}{dx}(xv)$ product rule

$= x\frac{dv}{dx} + v$ left side of DE

right side is $1 + v + v^2$

new DE in v: $x\frac{dv}{dx} + v = 1 + v + v^2$

$x\frac{dv}{dx} = 1 + v^2$ separable!

$\frac{1}{1+v^2}\,dv = \frac{1}{x}\,dx$

integrate

$\tan^{-1}(v) = \ln|x| + C$

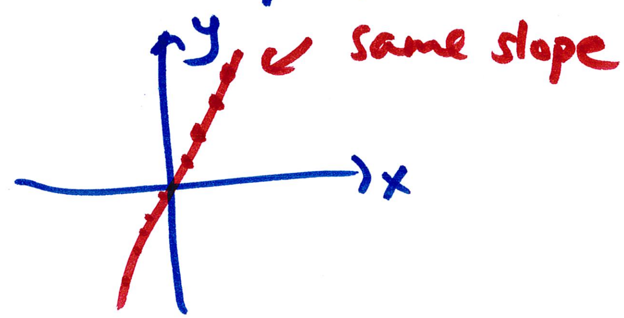
$$v = \tan(\ln|x| + c) \qquad y = xv$$

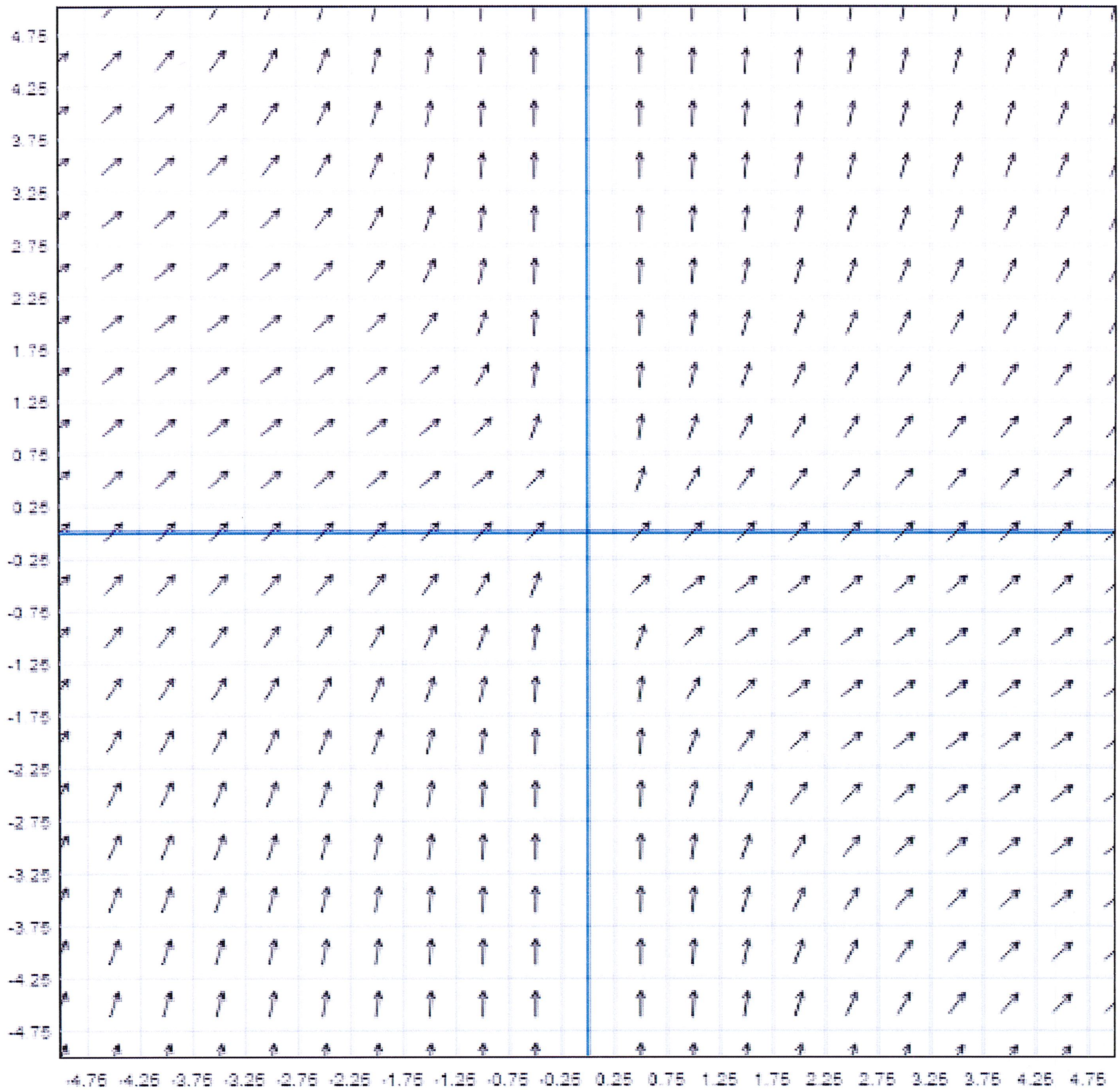
so $\boxed{y = x\tan(\ln|x| + c)}$

homogeneous DE: $\dfrac{dy}{dx} = f\left(\dfrac{y}{x}\right)$

so slope of y only depends on $\dfrac{y}{x}$

for example, slope at $(2,2)$ and $(4,4)$ are the same





any line thru origin will have same slope