1.1 Basic Models and Direction Fields

a differential equation (DE) is an equation that contains derivatives.

examples:

$$\left.\begin{aligned} &\frac{dy}{dt} = 2y + 3t \\ &y' = y(4-y) \\ &y'' + 3y' + y = 0 \end{aligned}\right\} \rightarrow \quad y = ?$$

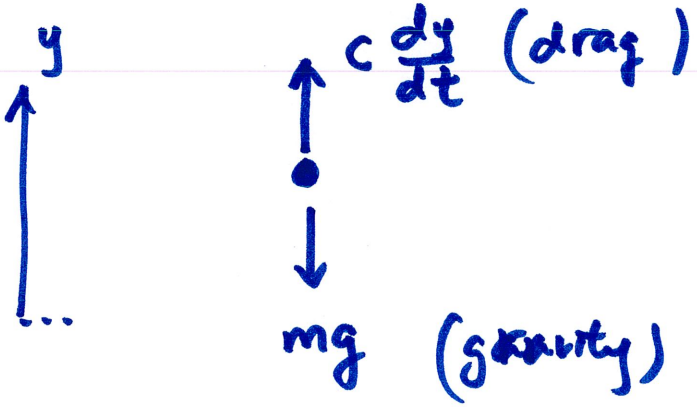
where do DEs come from in application?

population: $\frac{dP}{dt} = rP$ ← population, ← rate

interest: $\frac{dm}{dt} = rm$

Newton's 2nd Law : $F = ma$

free fall :



$$m\frac{d^2y}{dt^2} = c\frac{dy}{dt} - mg \quad \Rightarrow y(t)$$

$$m\frac{dv}{dt} = cv - mg \quad \Rightarrow v(t)$$

(v → velocity)

Slope / Direction Field

- a way to visualize solutions qualitatively.

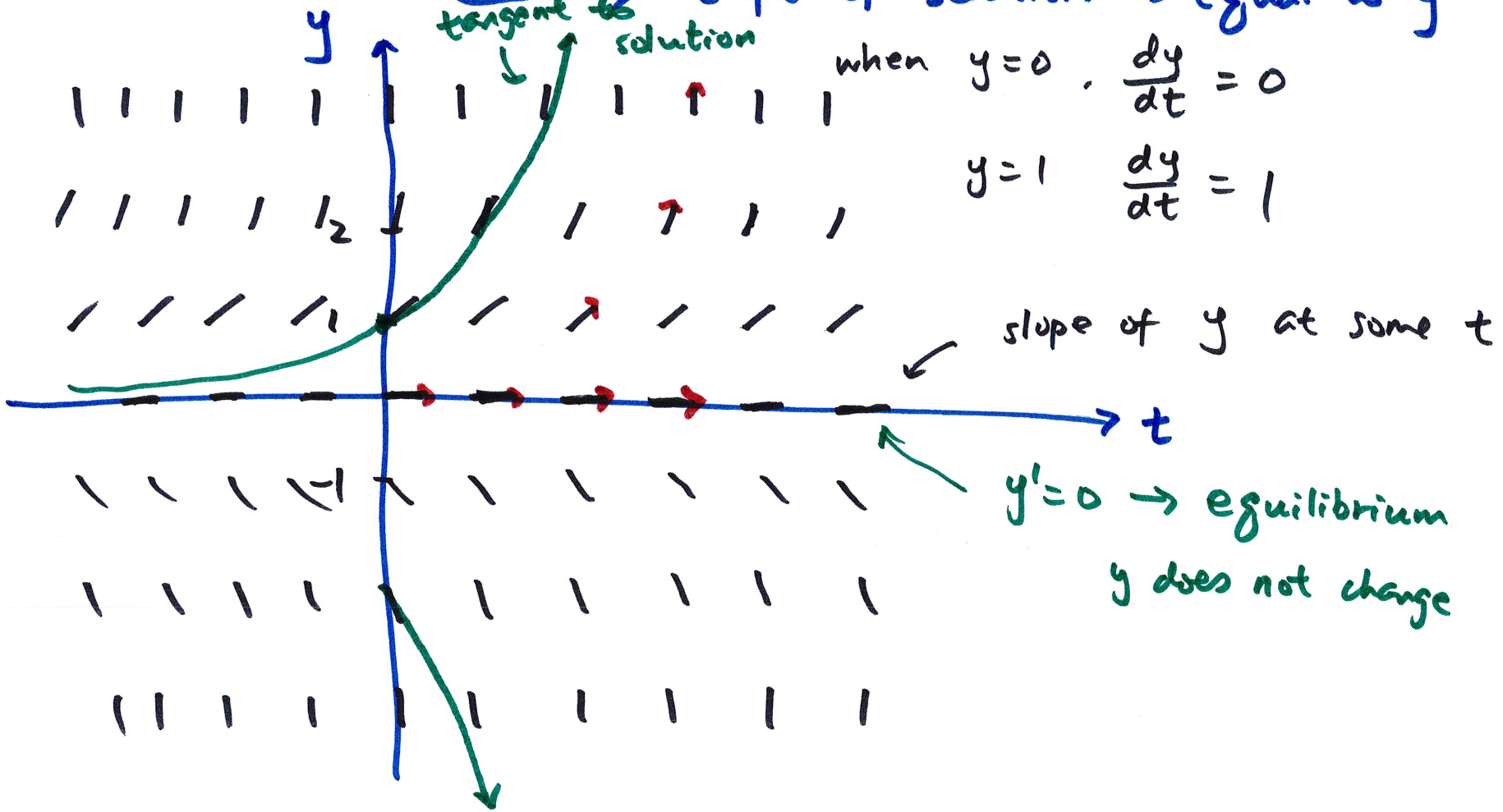
$\frac{dy}{dt}$: slope of $y(t)$

$\frac{dy}{dt} = y$

slope of solution is equal to y

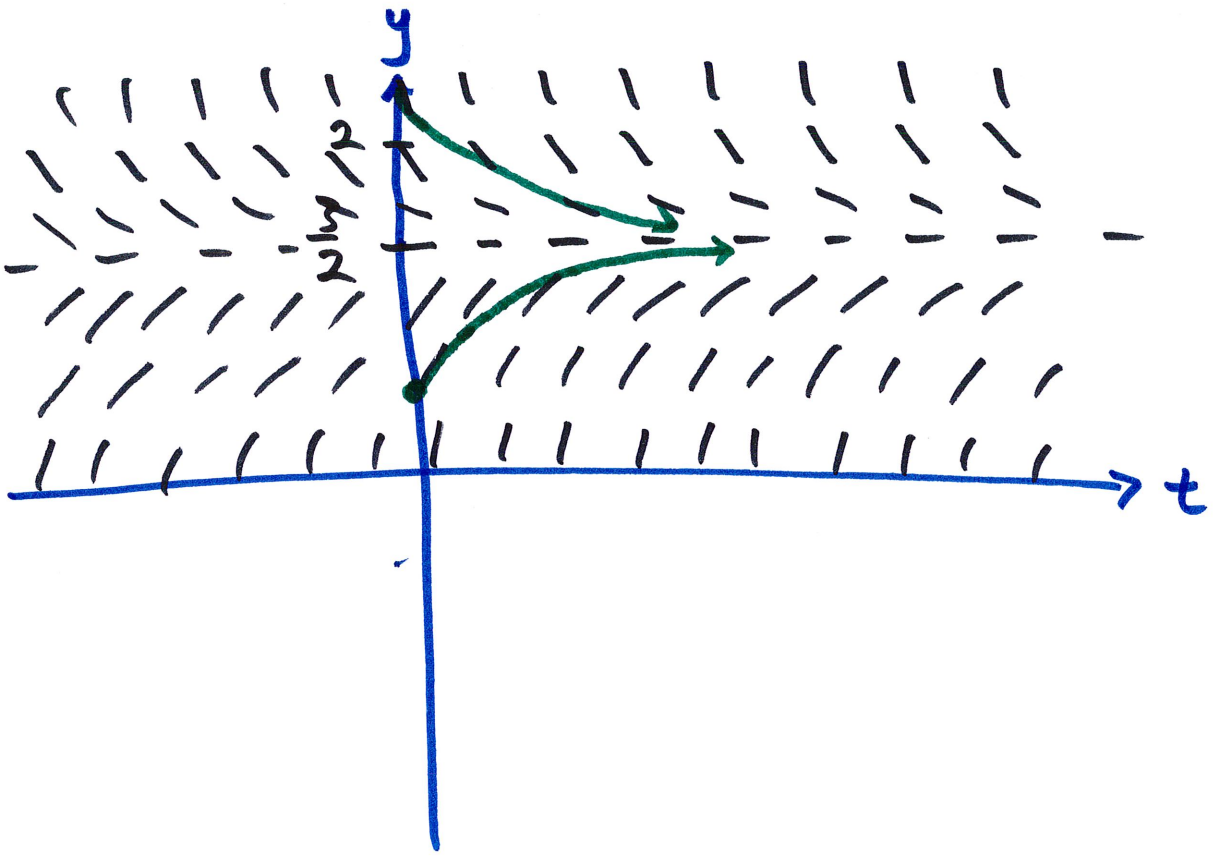
when $y=0$, $\frac{dy}{dt} = 0$

$y=1$ $\frac{dy}{dt} = 1$



example $y' = 3 - 2y$

find equilibrium: $y' = 0 \rightarrow y = \frac{3}{2}$



initial condition

if $y(0) = \frac{3}{2}$

as $t \to \infty$, $y \to \frac{3}{2}$

if $y(0) > \frac{3}{2}$

as $t \to \infty$, $y \to \frac{3}{2}$

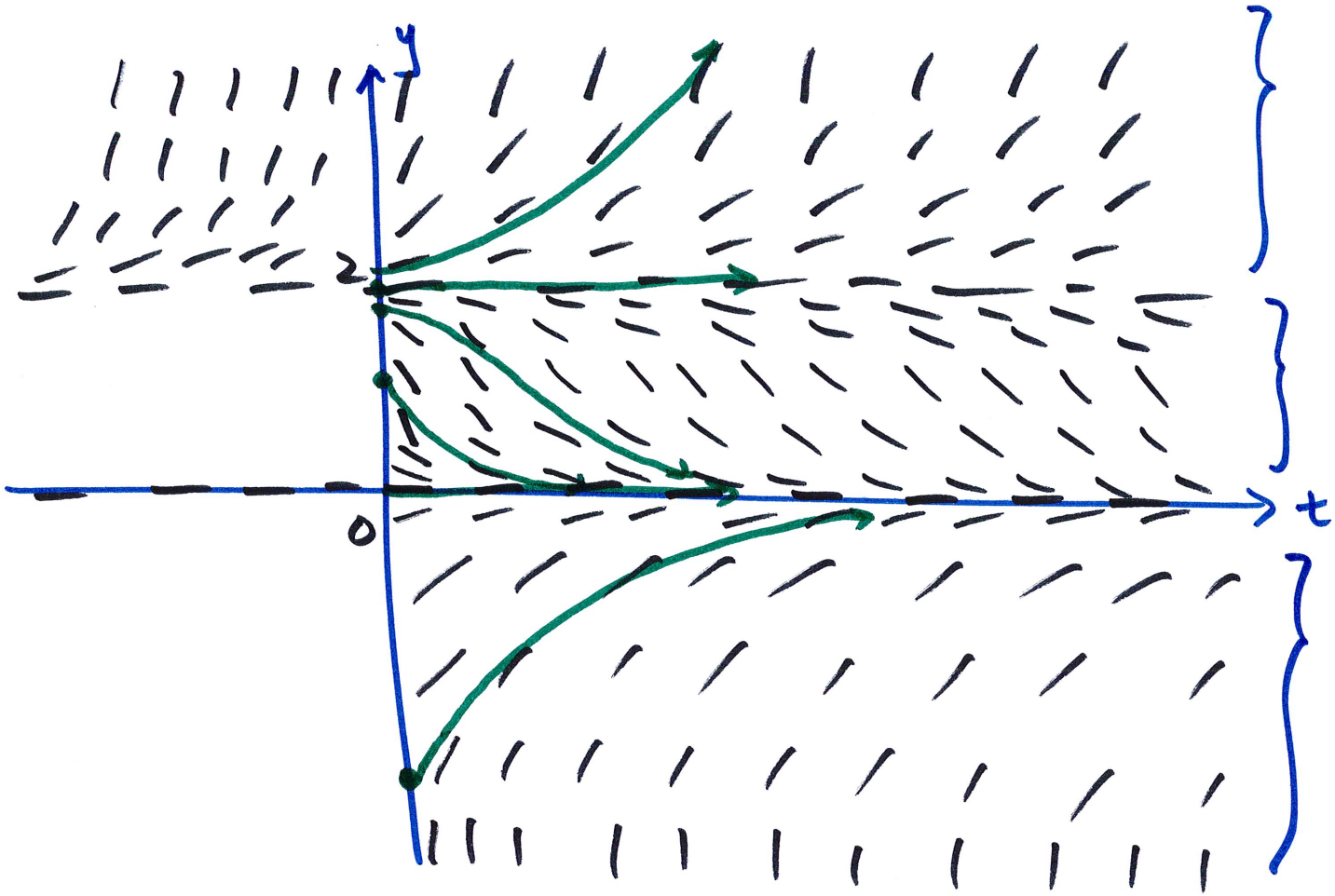
if $y(0) < \frac{3}{2}$

as $t \to \infty$, $y \to \frac{3}{2}$

all solutions converge to $y = \frac{3}{2}$ as $t \to \infty$

example $y' = y(y-2)$

equilibria : $y=0$, $y=2$



$y > 2$

$y' > 0$

grows as y increases

$0 < y < 2$

$y' < 0$

y' small near $y=2$ and $y=0$

$y < 0$

$y' > 0$

steep as y decreases

dfield 8 from Matlab

or

http://comp.uark.edu/~aeb019/dfield.html

zero