22. $y' = \dfrac{-t + \sqrt{t^2+4y}}{2}$ $\quad y(2) = -1$

$y_1 = 1-t \qquad y_2 = -t^2/4$

b). $y' = f(t,y) \quad y(t_0) = y_0$

has unique solution near (t_0, y_0)

if f and $\dfrac{\partial f}{\partial y}$ are both continuous

$f = \dfrac{-t + \sqrt{t^2+4y}}{2}$ at $(2,-1)$

$= \dfrac{-2 + \sqrt{4-4}}{2} = -1$ is continuous

$$\frac{\partial f}{\partial y} = \frac{\partial}{\partial y}\left[-\frac{1}{2}t + \frac{1}{2}(t^2+4y)^{1/2}\right] = \frac{1}{\cancel{4}}(t^2+4y)^{-1/2}(\cancel{4})$$

$$= \frac{1}{\sqrt{t^2+4y}}$$ DNE at $(2,-1)$

D.

$$\frac{dy}{dt} = y^2 - 4y \qquad y(0) = 8$$

largest interval where solution is defined?

$$\frac{dy}{dt} = y(y-4)$$

$$\frac{1}{y(y-4)} = \frac{A}{y} + \frac{B}{y-4}$$

$$\frac{1}{y(y-4)}\,dy = dt$$

$$1 = A(y-4) + B(y)$$

$$0y + 1 = (A+B)y - 4A$$

$$A+B=0 \rightarrow B = -A = \frac{1}{4}$$

$$1 = -4A \rightarrow A = -\frac{1}{4}$$

$$\int \left(-\frac{1}{4}\frac{1}{y} + \frac{1}{4}\frac{1}{y-4}\right) dy = \int dt$$

$$-\frac{1}{4}\ln|y| + \frac{1}{4}\ln|y-4| = t + K$$

$$\ln|y| - \ln|y-4| = -4t + a$$

$$\ln\left|\frac{y}{y-4}\right| = -4t+a$$

$$\frac{y}{y-4} = e^{-4t+a} = Ce^{-4t} \qquad y(0)=8$$

$$\frac{8}{4} = C = 2$$

$$\frac{y}{y-4} = 2e^{-4t}$$

$$y = 2e^{-4t}y - 8e^{-4t}$$

$$y - 2e^{-4t}y = -8e^{-4t}$$

$$y(1-2e^{-4t}) = -8e^{-4t}$$

$$y = \frac{-8e^{-4t}}{1-2e^{-4t}}$$

$$1-2e^{-4t} = 0$$

$$2e^{-4t} = 1$$

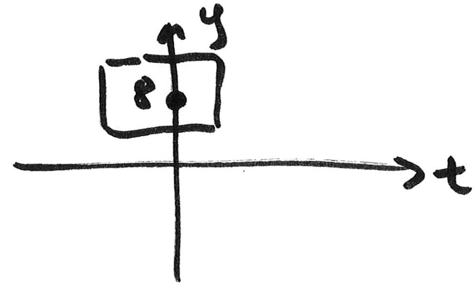
$$e^{-4t} = \frac{1}{2}$$

$$t = \dots$$

DE: $\quad \underbrace{y' = y^2 - 4y}_{f(t,y)} \qquad y(0) = 8$

$f(t,y)$ continuous for all y

$\frac{\partial f}{\partial y} = 2y - 4$ " " " "



2.5 Autonomous Equations

in the form $\frac{dy}{dt} = f(y)$

f does not depend on t
slope does not depend on t

e.g. population: $\frac{dy}{dt} = ry$

always separable whether linear or nonlinear

~~th~~ when $\frac{dy}{dt} = 0$ $\rightarrow$ equilibrium solutions or critical points

stability? (do solutions converge to or diverge from them?)

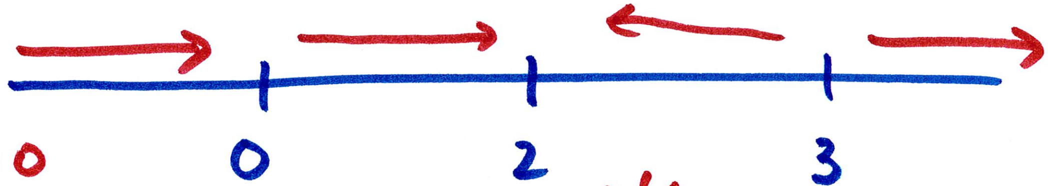
<u>example</u> : $\dfrac{dy}{dt} = y^2(y-2)(y-3)$

equilibrium solutions: $\dfrac{dy}{dt} = 0$

$y = 0, \quad y = 2, \quad y = 3$

check sign of $\dfrac{dy}{dt}$ between/outside these

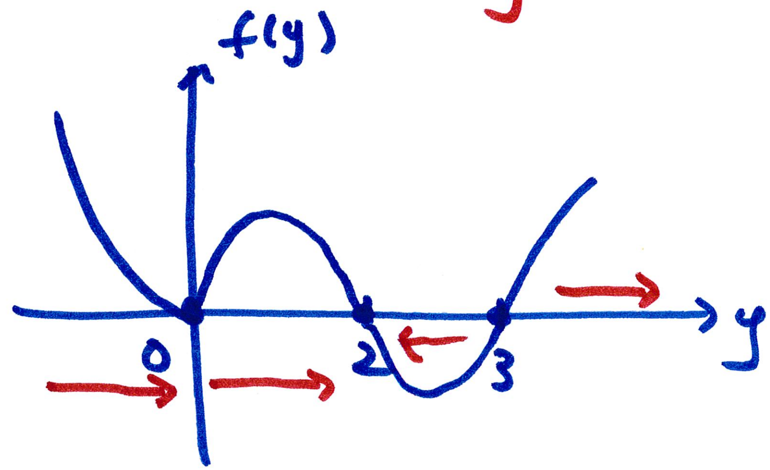
$y' = f(y)$ $+$ 0 $+$ 0 $-$ 0 $+$



$y' > 0$
y is increasing
moves to right

$y' < 0$
y is decreasing

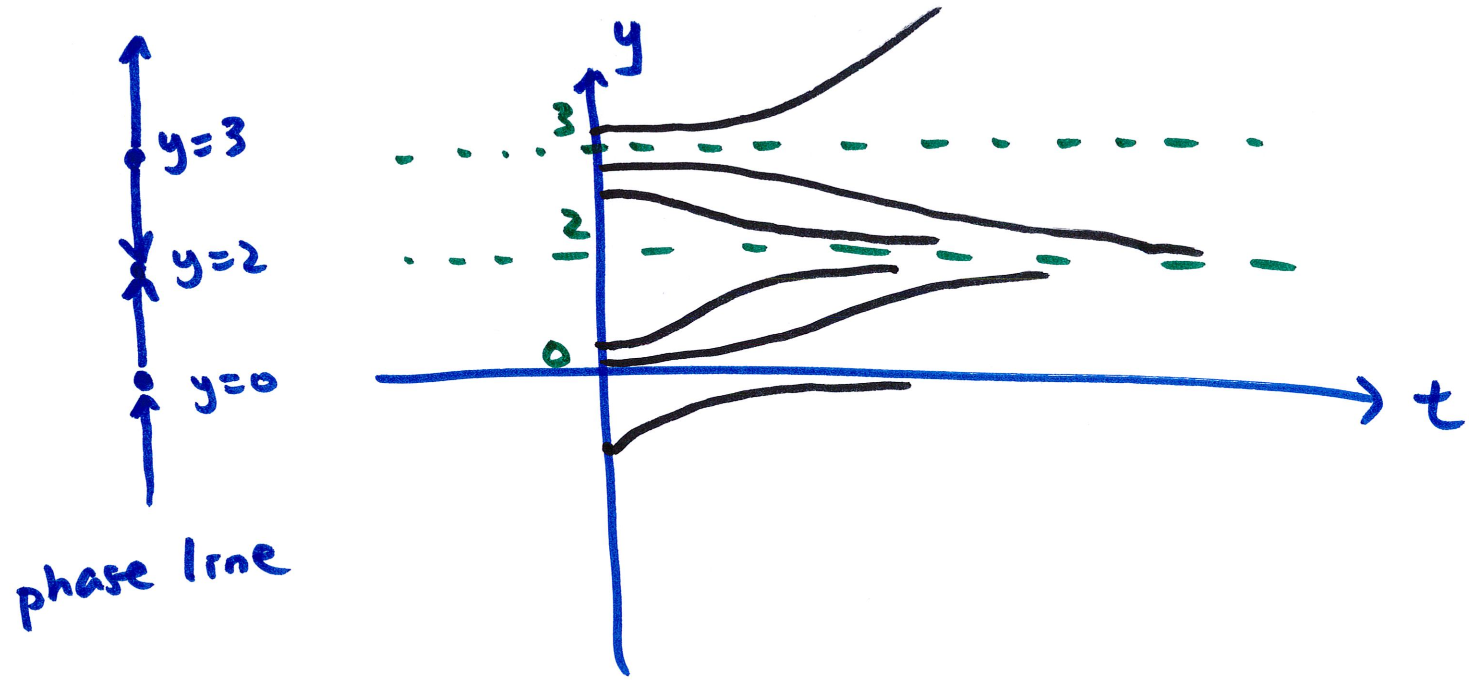
graph of $f(y)$ vs. y



$y=2$ is asymptotically stable because it attracts nearby solutions

$y=3$ is unstable because solutions diverge from it

$y=0$ is semistable because it is stable from one side, unstable from the other



<u>example</u> Gompertz equation $\dfrac{dy}{dt} = ry\ln\left(\dfrac{K}{y}\right)$ $y > 0$

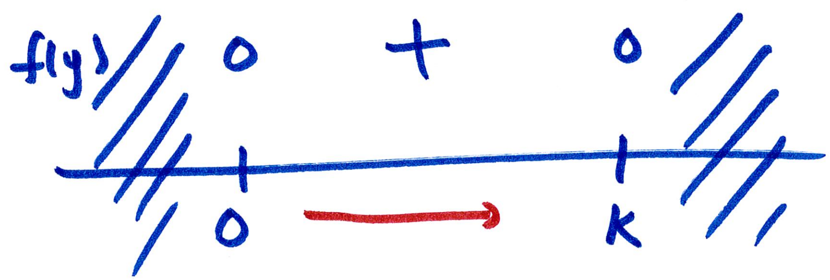
r : growth rate w/o limiting factors

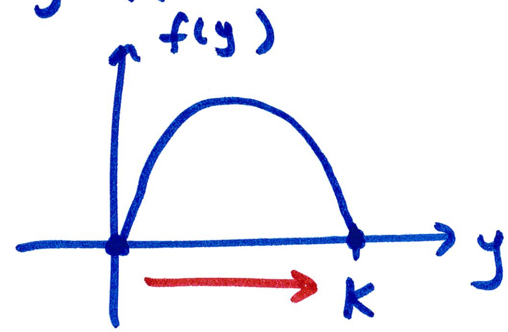
K : saturation level

→ used to model population in confined spaces (limited resources)

growth rate is slow at beginning and end

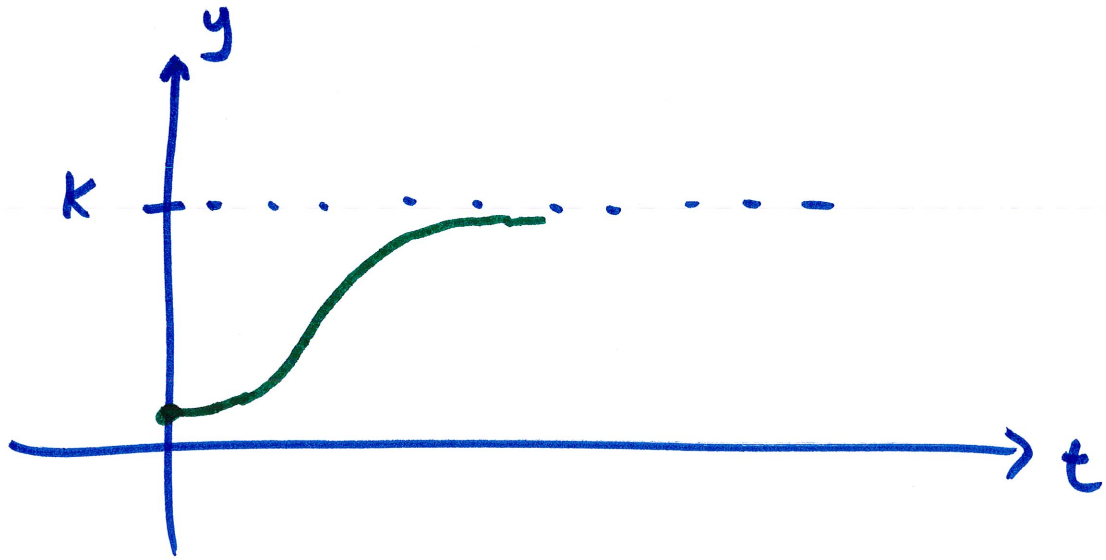
~~equil~~ critical points: $y = 0$, $y = K$





stability: $y = 0$ unstable

$y = K$ asymp. stable


y
K
t