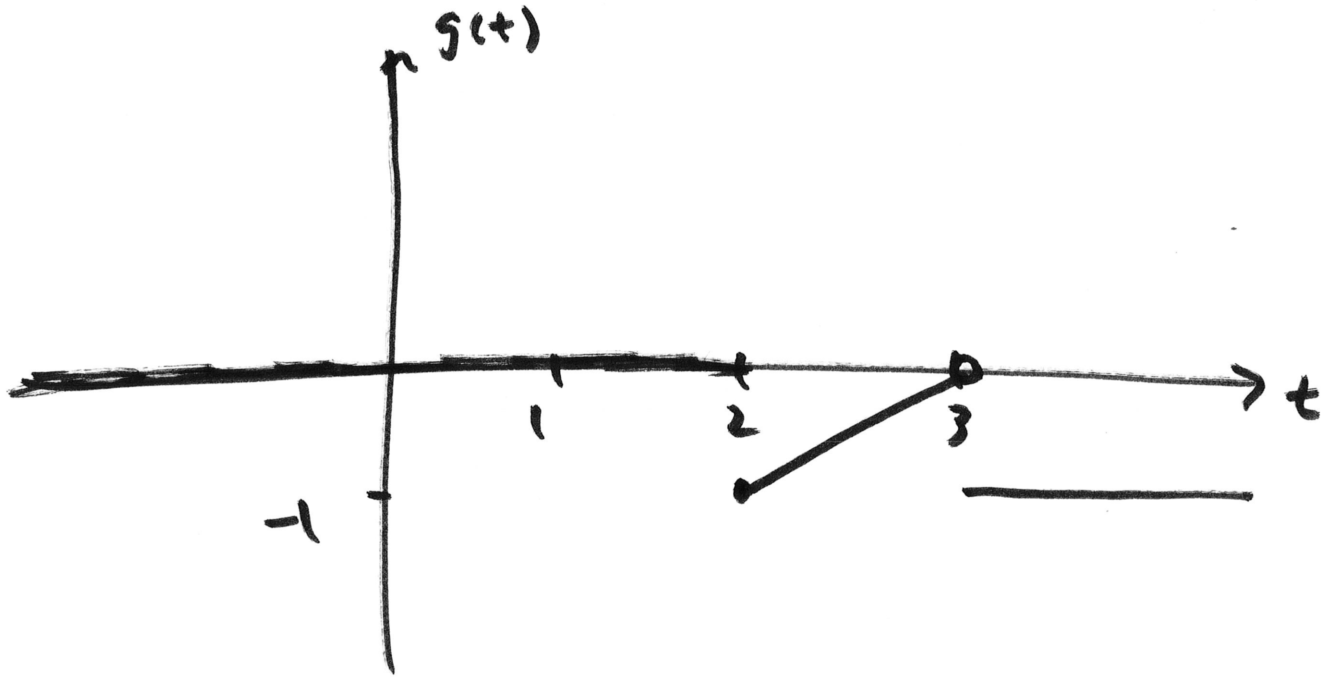
2. $g(t) = (t-3)u_2(t) - (t-2)u_3(t)$

$$= \begin{cases} 0 & t < 2 \\ t-3 & 2 \le t < 3 \\ t-3-(t-2) = -1 & t \ge 3 \end{cases}$$



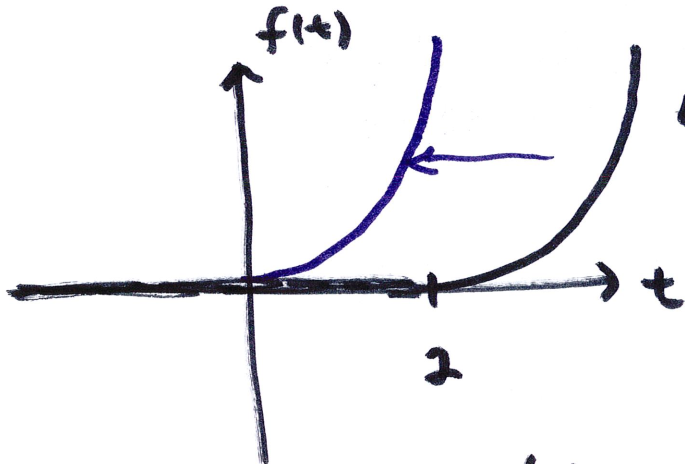
13. $f(t) = \begin{cases} 0 & t < 2 \\ (t-2)^2 & t \geq 2 \end{cases}$

LT of $f(t)$

t^2

$\mathcal{L}\{u_c(t)\, f(t-c)\} = e^{-cs}\, \mathcal{L}\{f(t)\}$

$(t-2)^2$



same function/shape but starting at $t = 0$

$(t-2)^2$ shift LEFT 2 units

$\rightarrow$ change t to $t+2$

$(t+2-2)^2 = t^2$

23. $$F(s) = \frac{(s-2)e^{-s}}{s^2-4s+3}$$

$$\mathcal{L}\{u_c f(t-c)\} = e^{-cs}\mathcal{L}\{f(t)\}$$

$$= e^{-s}\frac{s-2}{(s-3)(s-1)}$$

$$\Rightarrow \frac{1}{2}e^{3t} + \frac{1}{2}e^{t}$$

shifted

$$= e^{-s}\left[\frac{1}{2}\frac{1}{s-3} + \frac{1}{2}\frac{1}{s-1}\right]$$

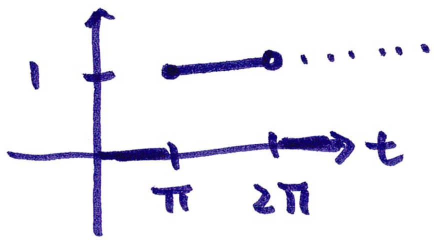
$t-1$

$$f(t) = \frac{1}{2}u_1(t)\left[e^{3(t-1)} + e^{\cancel{t(t-1)}}\right]$$

$$\cosh x = \frac{e^{x}+e^{-x}}{2}$$

6.4 DE's with Discontinuous Forcing Functions

example $\quad y'' + y = \begin{cases} 1 & \pi \le t < 2\pi \\ 0 & 0 \le t < \pi, \ t \ge 2\pi \end{cases}$



$y(0) = 0, \quad y'(0) = 1$

express right side with step functions

$$y'' + y = u_\pi(t) - u_{2\pi}(t)$$

$$s^2 Y - \cancel{s y(0)} - y'(0) + Y = \frac{e^{-\pi s}}{s} - \frac{e^{-2\pi s}}{s}$$

$$(s^2+1)Y = e^{-\pi s}\left(\frac{1}{s}\right) - e^{-2\pi s}\left(\frac{1}{s}\right) + 1$$

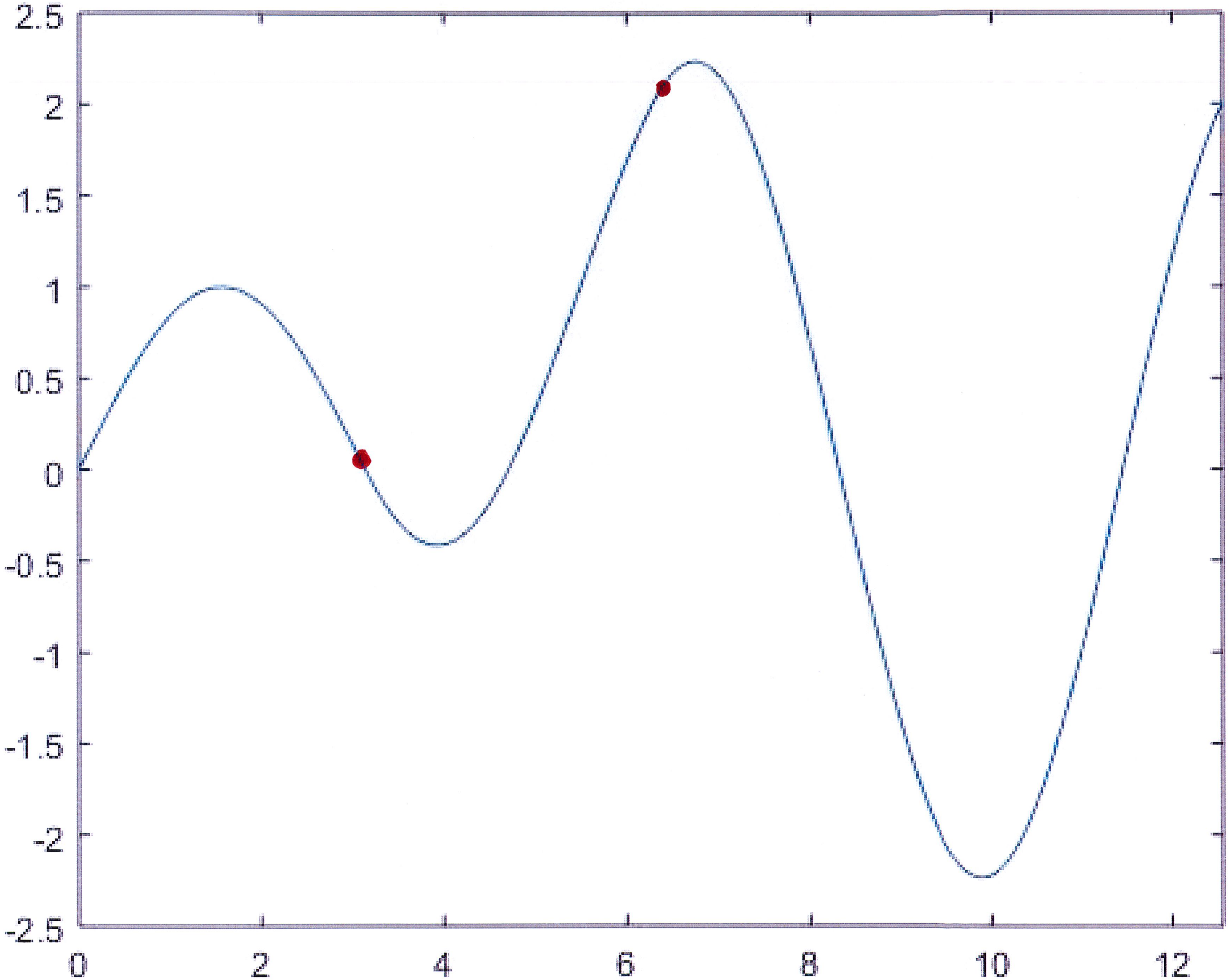
$$Y = e^{-\pi s}\frac{1}{s(s^2+1)} - e^{-2\pi s}\frac{1}{s(s^2+1)} + \frac{1}{s^2+1}$$

$$Y = \frac{1}{s^2+1} + e^{-\pi s}\underbrace{\left(\frac{1}{s} - \frac{s}{s^2+1}\right)}_{1-\cos(t)} - e^{-2\pi s}\left(\frac{1}{s} - \frac{s}{s^2+1}\right)$$

$$y(t) = \sin(t) + u_\pi(t)\left[1 - \underbrace{\cos(t-\pi)}_{-\cos(t)}\right] - u_{2\pi}(t)\left[1 - \underbrace{\cos(t-2\pi)}_{\cos(t)}\right]$$

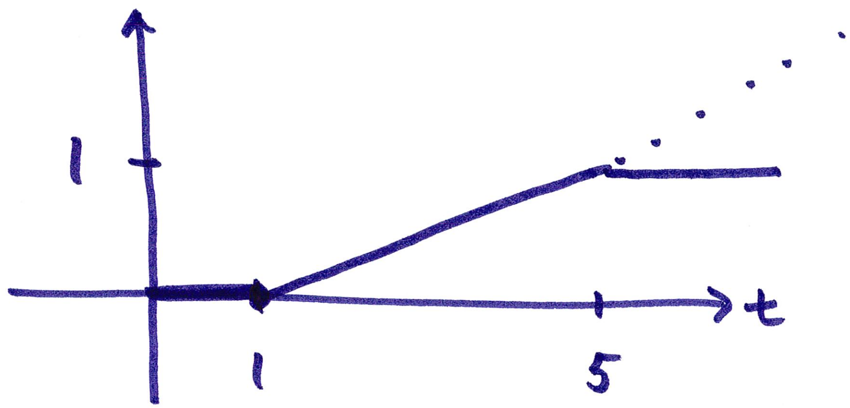
$$\boxed{y = \sin(t) + u_\pi(t)(1+\cos t) - u_{2\pi}(t)(1-\cos t)}$$

$$= \begin{cases} \sin(t) & t < \pi \\ \sin(t) + 1 + \cos(t) & \pi \le t < 2\pi \\ \sin(t) + 1 + \cos(t) \\ \quad -1 + \cos(t) & t \ge 2\pi \\ = \sin(t) + 2\cos(t) & \end{cases}$$


2.5
2
1.5
1
0.5
0
-0.5
-1
-1.5
-2
-2.5
0
2
4
6
8
10
12

example

$$y'' + y = \begin{cases} 0 & 0 \le t < 1 \\ \frac{t-1}{4} & 1 \le t < 5 \\ 1 & t \ge 5 \end{cases} \qquad y(0) = y'(0) = 0$$



right side with step function

$$y'' + y = u_1(t)\left(\frac{t-1}{4}\right) + u_5(t)\left(-\frac{t-1}{4} + 1\right)$$

cancels this (back to 0)

target at 1

$$y'' + y = u_1(t)\left(\frac{t-1}{4}\right) + u_5(t)\left(\frac{5}{4} - \frac{t}{4}\right)$$

$$= \frac{1}{4} u_1(t)(t-1) + \frac{1}{4} u_5(t)(5-t)$$

$$y'' + y = \frac{1}{4} u_1(t)(t-1) - \frac{1}{4} u_5(t)(t-5)$$

shift then LT

t shifted LEFT 1 unit

$$(s^2+1)Y = \frac{1}{4} e^{-s}\left(\frac{1}{s^2}\right) - \frac{1}{4} e^{-5s}\left(\frac{1}{s^2}\right)$$

$$Y = \frac{1}{4} e^{-s} \frac{1}{s^2(s^2+1)} - \frac{1}{4} e^{-5s} \frac{1}{s^2(s^2+1)}$$

$$= \frac{1}{4} e^{-s}\left(\frac{1}{s^2} - \frac{1}{s^2+1}\right) - \frac{1}{4} e^{-5s}\left(\cancel{\frac{1}{s^2}}\ \frac{1}{s^2} - \frac{1}{s^2+1}\right)$$

$t - \sin t$

now shift RIGHT

$$y = \frac{1}{4}u_1(t)\left[(t-1) - \sin(t-1)\right] - \frac{1}{4}u_5(t)\left[(t-5) - \sin(t-5)\right]$$

$$= \begin{cases} 0 & t < 1 \\ \dfrac{t-1-\sin(t-1)}{4} & 1 \le t < 5 \\ \dfrac{4-\sin(t-1)+\sin(t-5)}{4} & t \ge 5 \end{cases}$$

MATLAB : "fplot" fplot ('function name', time interval)