MA 266 Lecture 4

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Population dynamics

Example 1. *According to a census, the world's total population reached 6 billion persons in mid-1999, and was then increasing at a* ***rate*** *of about 212 thousand persons each day. Assuming that natural population growth at this rate continues:*

a) What is the annual growth rate k ?

Let $P(t)$ denote the population in billions and let t be the time in years. Then, the differential equation describing the population dynamics is:

$$\frac{dP}{dt} = \underbrace{(b-d)}_{=:k} P(t). \tag{1}$$

Set $t = 0$ for mid-1999. Then, the initial population is $P(0) = 6$ billion. Observe that the instantaneaous increase of population is

$$212 \text{ [thousand persons / day]} \equiv 0.000212 \text{ [billion persons / day]}$$

Hence, the efective increase at time $t = 0$ is:

$$P'(0) = 0.000212 \text{ [billion / day]} \cdot 365.25 \text{ [days / year]} \approx 0.07743 \text{ [billion / year]}.$$

Using the above result, we can compute k as follows:

$$k = \frac{P'(0)}{P(0)} \approx \frac{0.07743}{6} = 0.0129.$$

The *particular solution* is then

$$P(t) = 6 \cdot e^{0.0129 \cdot t} \tag{2}$$

b) What would be the population at the middle of the 21st century?

We use the particular solution (2) to make preditions:

$$P(t = 51 \text{ years}) = 6 \cdot e^{0.0129 \cdot 51} \approx 11.58 \text{ [billions]}$$

c) How long will it take the world to increase tenfold –thereby reaching 60 billion that some demographers believe to be the maximum for which the planet can provide food supplies?

Solve for t, the folowing equation:

$$60 = P(t) = 6 \cdot e^{0.0129 \cdot t}$$

The obtained time t is:

$$t = \frac{\ln(10)}{0.0129} \approx 178 \text{ [years]},$$

i.e., the population will reach 60 billion in the year 2177.