MA 266 Review Midterm

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Review Midterm - Examples taken from previous exams

Example 1. *Determine the interval where the solution guaranteed to exist for the following IVP:*

$$(t+2)y' + y = \frac{1}{t-1}, \qquad y(0) = \frac{1}{2}.$$

Example 2. *Find the general solution of*

$$x^2y' + 2xy = e^{3x}.$$

Example 3. *If $y = y(x)$ is the solution to*

$$\frac{dy}{dx} = \frac{4xy}{2+x^2}, \qquad y(0) = 4,$$

then $y(\sqrt{3}) =$?

Example 4. *Let $y(t)$ denote the solution to the IVP:*

$$y''' + 3y'' + 2y' = 0, \quad y(0) = 2,\ y'(0) = -1,\ y''(0) = 1.$$

What is the value of $y(1)$?

Example 5. *Solve the initial value problem for the homogeneous equation:*

$$\frac{dy}{dx} = \frac{x^4 + y^4}{xy^3}, \quad y(e) = e, \quad x > 0.$$

by using a substitution $v = y/x$.

Example 6. *Find if the following equation is exact:*

$$(e^x \sin y - 2y \sin x) + (e^x \cos y + 2\cos x + 2y)y' = 0; \quad y(0) = \pi.$$

If YES, find the implicit solution of the IVP with initial condition $y(0) = \pi$.

Example 7. *Determine the stability of the equilibrium solutions of the following autonomous equation:*

$$\frac{dy}{dt} = y(9 - y^2).$$

Example 8. *Use Euler's method to find approximate value of* $y(0.2)$ *for the following IVP with step size* $h = 0.1$,

$$y' = y^2 + t^2, \quad y(0) = 1.$$

Example 9. *Consider a pond that initially contains 10 million gal of water. Water containing a polluted chemical flows into the pond at the rate of 6 million gal/yr, and the mixture in the pond flows out at the rate of 5 million gal/yr. The concentration $\gamma(t)$ of chemical in the incoming water varies as $\gamma(t) = 2 + \sin 2t$ grams/gal. Let $Q(t)$ be the amount of chemical at time t measured by millions of grams. Derive the differential equation of the process.*

Example 10. *Let $y(t)$ be the solution of the IVP:*

$$y''' + y' = 0, \quad y(0) = 2, \quad y'(0) = 1, \quad y''(0) = 1,$$

then $y(\pi) = ?$

Example 11. *Find the particular solution of the IVP:*

$$y' = \frac{1-2x}{y}, \quad y(1) = -2,$$

in explicit form.

Example 12. *Find the solution of the IVP*

$$y'' + y' - 6y = 0, \quad y(0) = 0,\ y'(0) = 5.$$