MA 266 Lecture 1

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Sec 1.1 Differential Equations; Mathematical Models

Question: What is a differential equation?

A **differential equation** is

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Example 1. ***(Types of equations)***

1. *Find x in* $x^2 + 6x + 1 = 0$.

2. *Find $f(t)$ in* $f(t)\cos(t) = e^t - \sin(t)$.

3. *Find $y(t)$ in* $y'' + 10y' = e^t$.

Question: Why do we study differential equations?

- Many natural phenomena; physical processes involve ______________.
- $\frac{dx}{dt} = f'(t)$ is the ______________ at which $x = f(t)$ is ____________________________.
- Differential equations to model ____________________________.

Example 2. ***(An example of mathematical model — object-spring)***

Consider an object with a mass m attached to the end of a spring. The mass experiences a force $F(t)$. Formulate a differential equation to model its motion.

- **Notations**

- **Physical Law: Newton's law**

- **Forces that acted on the object**

Remark The differential equation contains two constants: m, and k

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Definitions

- The **order** of a differential equation is the **order** of the highest ____________ involved in the ODE.

Example 3. ***(Find the order)***

1. $4x^2y'' + y = 0$

2. $(y')^2 + y^2 = -1$

3. $y^{(3)}x^2 + x^{10}y = \sin(x)$

- The **general** form of an n-**th order** differential equation:

- We say ____________ is a **solution** of the differential equation ____________

- **Initial value problem** (IVP): ____________ together with an ____________.

- The solution to an ODE for ____________ is called **particular** solution.

- **General solution**
  - Without an ____________, the ODE may have ____________ solutions.
  - If we can write an expression for ____________ solution $\equiv$ **general solution**.

Example 4. ***(Population Dynamics)***
Consider the time rate of change of a population $P(t)$.

- **Notation**
  - constant birth rate ____________
  - constant death rate ____________

- **Differential equation**

1. Check $P(t) = Ce^{kt}$ is a general solution

2. Suppose that the population at time $t = 0$ (hours, h) was 1000. Find the value of C

3. Assume the population doubled after 1 hour, determine the value of k

4. Write the particular solution. Use it to predict the population after 1.5 hours

- **Ordinary** differential equations (ODE): the ____________ ____________ depends on a ____________ ____________ variable.
- **Partial** differential equations (PDE): If the ________________________ is a function of ____________ ____________ variables.

Example 5. ***(Thermal Diffusivity)***
Consider a one dimensional rod. The temperature ____________ *satisfies the heat equation:*

where ____________ is the thermal diffusivity.