MA 266 Lecture 2

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Sec 1.2 Integrals as General/Particular Solutions

In this section, we discuss how to solve differential equations.

- Consider the **first order** equation:

$$\frac{dy}{dx} = f(x, y)$$

- Consider the simple case

$$\frac{dy}{dx} = f(x)$$

Example 1. *Find the solution $y(x)$ of the simple case:*

Remarks

- ____________ is the ____________ solution.
- Involves an arbitrary constant ____________.
- For every choice of ____________, ____________ is a solution of ____________.

- Consider the **Initial Value Problem** (IVP):

$$\frac{dy}{dx} = f(x), \qquad y(x_0) = y_0.$$

Example 2. *Find the* ***particular*** *solution of the IVP.*

Example 3. *Find the particular solution $y(x)$ of the following IVP.*

$$\frac{dy}{dx} = \sin(x), \qquad y(0) = 1.$$

Second Order Equations

- Consider the second-order differential equation of the special form:

$$\frac{d^2y}{dx^2} = g(x)$$

Example 4. *Find the general solution of this second-order equation.*

Remark

- The above second-order differential equation can be solved by solving successively the

  ____

Velocity and acceleration

Notation

- The motion of a particle along a straight line (the x-axis) is described by its position function:

- ____________ is the x-coordinate at time t.

- The **velocity** $v(t)$ of the particle is:

- The **acceleration** $a(t)$ is:

Example 5. *Find the general solution when the acceleration is constant $a(t) = a$.*

Example 6. *Given an initial position $x(0) = x_0$ and initial velocity $v(0) = 0$, find the particular solution of the corresponding IVP.*

Example 7. *At 12:00 PM, a car starts from rest at point A and proceeds at constant acceleration along a straight road towards point B. The car reaches B at 12:50 PM with velocity of* 60 *miles/hour. Find the distance from A to B.*

Vertical Motion and Gravitational Acceleration

Notation

- The weight ____________ of a body is the force exerted on the body by gravity.

- If we ignore the air resistance, then the acceleration ____________ is

- The **velocity** equation is:

- The **height** equation is:

Example 8. *Suppose that a ball is thrown straight upward from the ground ($y_0 = 0$) with initial velocity $v_0 = 96$ ft/s (then $g = 32$ ft/s^2). Find the maximum height the ball attains.*

A Swimmer's Problem

Consider a northward-flowing river of width $w = 2a$. The lines $x = \pm a$ represent the banks of the river and the y-axis its center. Suppose that the velocity v_R at which the water flows increases as one approaches the center of the river. v_R is given by

$$v_R = v_0 \left(1 - \frac{x^2}{a^2}\right).$$

Example 9. *Suppose that a swimmer start at point $(-a, 0)$ on the west bank and swims due east (relative to the water) with constant speed v_S. His velocity (relative to the riverbed) has a horizontal component v_S and a vertical component v_R. Find the swimmer trajectory $y(x)$.*