MA 266 Lecture 3

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Sec 1.3 Slope Fields and Solution Curves

Consider the first order differential equation:

$$\frac{dy}{dx} = f(x, y)$$

Question: Can we find the solution $y(x)$ using

$$y(x) = \int f(x, y(x))dx + C \quad ?$$

Answer: ____________.

- ____________ involves the _________________________.

Slope fields and Graphical solutions

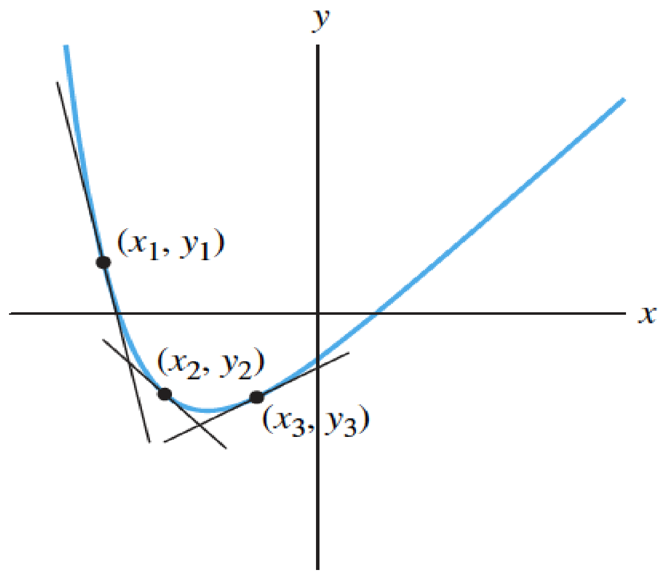
- For each ____________, ____________ determines _________________________.

Definition. _________________________ of the differential equation $\frac{dy}{dx} = f(x, y)$ is a

_________________________ whose ____________ at each ____________ has:

Example 1. *Consider* ***a*** *solution curve of*

$$\frac{dy}{dx} = x - y$$



- point (x_1, y_1)
- point (x_2, y_2)
- point (x_3, y_3)

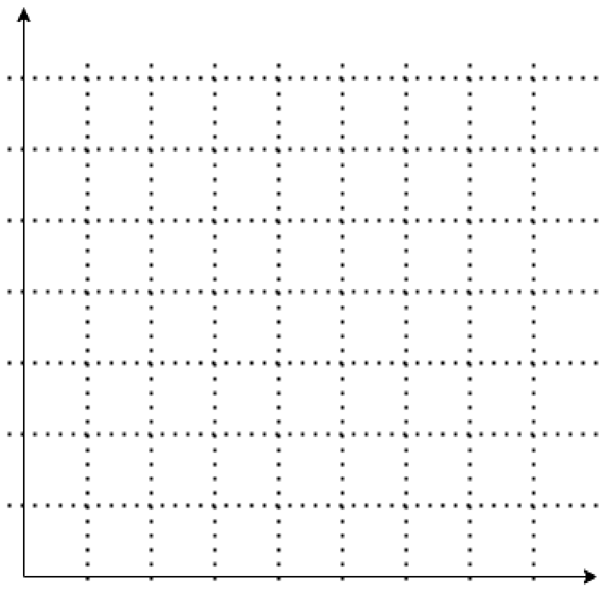
Constructing slope fields.

- Consider a representative collection of points __________________________.
- For each _____________, we draw a "short" line segment having:
- The collection of line segments: _________________________________.

Example 2. *Consider the population dynamics:*

$$\frac{dP(t)}{dt} = kP(t).$$

Construct the slope field.



- Recall the general solution:

- How do we draw the tangent line?

Remark

- We use he slope field to study the ____________ of ________________________.

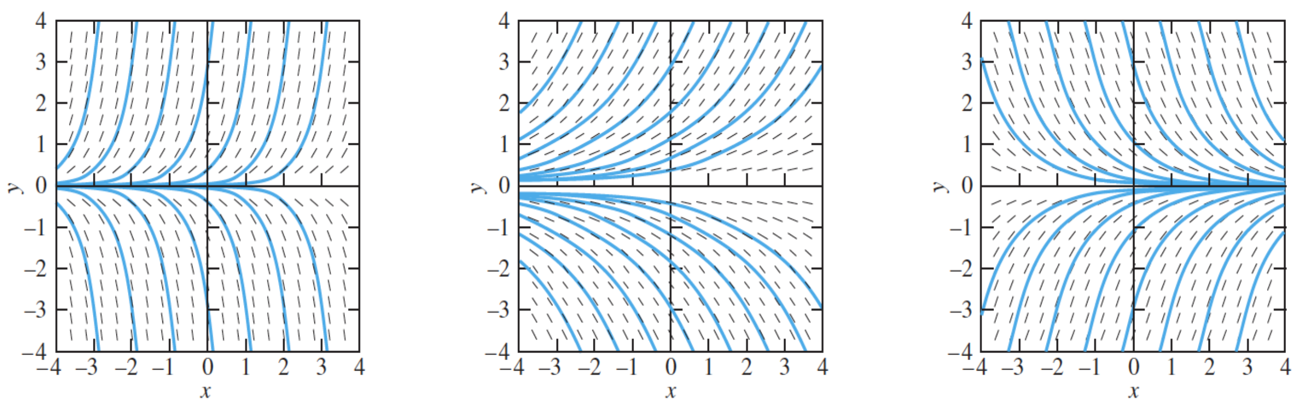
Example 3. *Consider the differential equation*

$$\frac{dy}{dx} = ky,$$

where ____________ *is the rate of change of* ____________.

- The general solution is:

- The solution curves and slope fields for $k = 2, 0.5, -1$:



Existence of Solutions

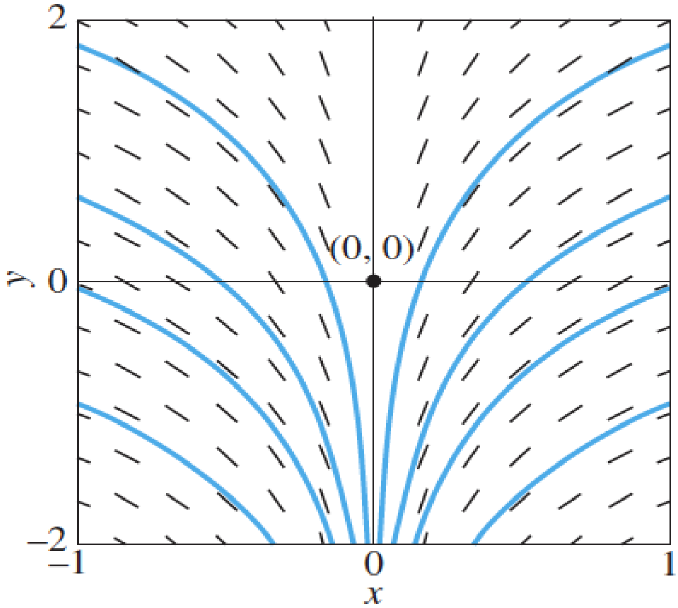
Example 4. *Consider the IVP:*

$$\frac{dy}{dx} = \frac{1}{x}, \qquad y(0) = 0.$$

- General solution:

- Particular solution:

The slope field:



Remark

- The ________________________ forces all curves near ____________ to plunge

  downward so that none can pass through ____________.

Uniqueness of Solutions

Example 5. *Consider the IVP:*

$$\frac{dy}{dx} = 2\sqrt{y}, \qquad y(0) = 0.$$

- Check if ____________ is a solution:

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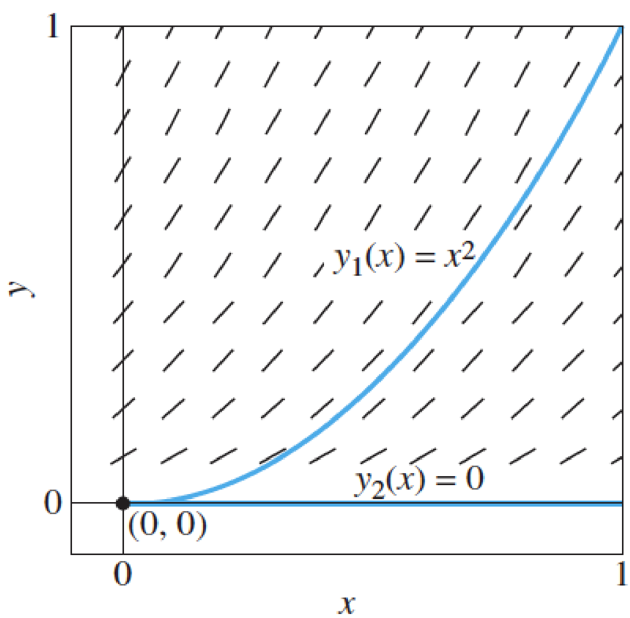


Theorem 1. *(Existence and Uniqueness of Solutions) Suppose that both the function $f(x, y)$ and its partial derivative $D_y f(x, y)$ are continuous on some rectangle R in the xy-plane that contains the point (a, b) in its interior. Then, for some open interval I containing the point a, the initial value problem*

$$\frac{dy}{dx} = f(x, y), \qquad y(a) = b$$

has ***one and only one solution*** *that is defined on the interval I.*

Example 6. *Consider*

$$\frac{dy}{dx} = 2\sqrt{y}$$

Example 7. *Consider:*

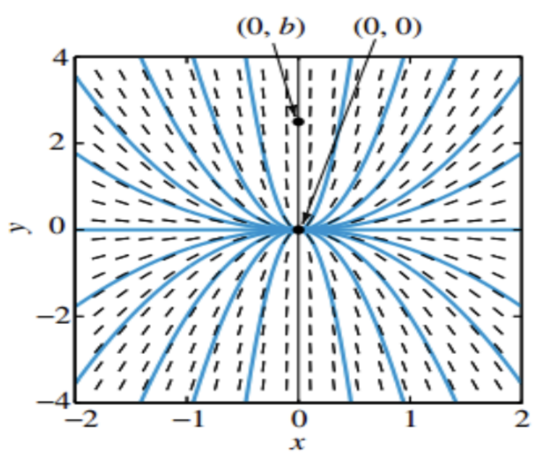
$$x\frac{dy}{dx} = 2y$$

a) Check the existence and uniqueness of the IVP:

$$x\frac{dy}{dx} = 2y, \quad y(0) = b.$$

- Case $b = 0$:

- Case $b \neq 0$:



b) Check the existence and uniqueness of the IVP:

$$x\frac{dy}{dx} = 2y, \quad y(a) = b, \quad a, b \neq 0.$$