MA 266 Lecture 4

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Sec 1.4 Separable Equations and Applications

Defintion 1. *We call a first-order differential equation* __________________ *if it can be written as:*

$$\frac{dy}{dx} = f(x, y) =$$

where __________________.

Example 1. *Determine* $g(x)$ *and* $h(y)$

1. *Consider*

$$x^3 \frac{dy}{dx} = e^{-y}$$

$$g(x) = \_\_\_\_\_\_\_\_\_\_ \quad \textit{and} \quad h(y) = \_\_\_\_\_\_\_\_\_\_.$$

2. *Consider*

$$\frac{dy}{dx} = \frac{x^{1000}}{y}$$

$$g(x) = \_\_\_\_\_\_\_\_\_\_ \quad \textit{and} \quad h(y) = \_\_\_\_\_\_\_\_\_\_.$$

3. *Consider*

$$\frac{dy}{dx} = 100 \cdot (xy)^{3/5}$$

$$g(x) = \_\_\_\_\_\_\_\_\_\_ \quad \textit{and} \quad h(y) = \_\_\_\_\_\_\_\_\_\_.$$

Solving Separable Equations

1. The separable equation:

$$\frac{dy}{dx} = \frac{g(x)}{h(y)}$$

2. Write in the form:

3. Integrate both sides:

4. We only need the ___________________:

    •

    •

Remark

- Equations _____________ and _____________ are equivalent:

Example 2. *Find the general solution of*

$$\frac{dy}{dx} = y\sin(x)$$

- For ____________________, separating variables gives:

- Integrating both sides:

Remark

- For _____________, _____________ has a _____________ solution.

Defintion 2. ____________________ *solutions are exceptional solutions that cannot be obtained by selecting a value for* _____________.

Example 3. *Solve the differential equation:*

$$\frac{dy}{dx} = \frac{4-2x}{3y^2-5}$$

- Separating variables:

- Is ____________ defined for all y? ____________.

- This implies that for ____________, ________________________ can cross either of the ____________ lines:

- Divides the __________________ into:

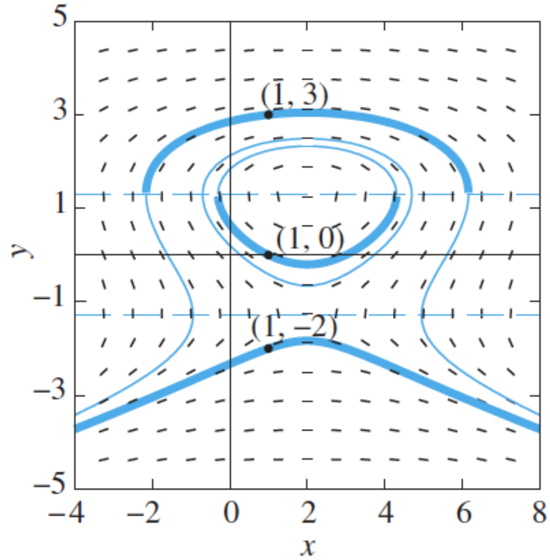
- Integrating $\implies$ general solution:

- **Question:** Can we solve the general solution for y? ____________.

- __________________ are contained in __________________ of:

Example 4. *Solve the initial value problem:*

$$\frac{dy}{dx} = \frac{4-2x}{3y^2-5}; \qquad y(1) = 3.$$



- Compute C of the general solution:

- __________________________ lies on the level curve:

Example 5. *Find all solutions of the differential equation:*

$$\frac{dy}{dx} = 6x(y-1)^{2/3}$$

- Separation of variables gives:

- Is __________________ a **singular** solution?

- Are a) $y(x) \equiv 1$ and b) $y(x) = 1 + (x^2 - 1)^3$ solutions of the IVP with IC: $y(1) = 1$?

Natural Growth and Decay

The differential equation:

$$\frac{dx}{dt} = kx, \qquad k \text{ a cosntant}$$

serves as a mathematical model for wide range of natural phenomena:

- Population dynamics
- Compound interest
- Radioactive decay

The **general** solution:

- Separating the variables and integrating:
- We solve for x:
- The particular solution for the IC $x(0) = x_o$ is:

Population dynamics

Example 6. *According to a census, the world's total population reached 6 billion persons in mid-1999, and was then increasing at a* ***rate*** *of about 212 thousand persons each day. Assuming that natural population growth at this rate continues:*

a) What is the annual growth rate k ?

b) What would be the population at the middle of the 21st century?

c) How long will it take the world to increase tenfold –thereby reaching 60 billion that some demographers believe to be the maximum for which the planet can provide food supplies?

Radioactive Decay

Consider a sample of material that contains $N(t)$ atoms of a certain radioactive isotope a time t. It has been observed that a constant fraction of those radioactive atoms will spontaneously decay during each unit of time. Consequently, the sample behaves exactly like the population dynamics with no births $b = 0$. The model for the $N(t)$ atoms is then

$$\frac{dN}{dt} = -kN(t).$$

Example 7. *An accident at a nuclear plant has left the surrounding area polluted with radioactive material that decays naturally. The initial amount of radioactive material is 15 (safe units), and 5 months later is still 10 su.*

- Write a formula given the amount $N(t)$ of radioactive material (in su) after t months.

- What amount of radioactive material will remain after 8 months?

- How long it will be until $N = 1$ su, so it is safe for people to return to the area?