MA 266 Lecture 5

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Sec 1.5 Linear First-Order Equations

Defintion 1. *A ____________________________________ is a differential equation of the form:*

$$\frac{dy}{dx} + P(x)y = Q(x).$$

The coefficients $P(x)$ and $Q(x)$ are assumed to be ***continuous*** *in some interval on the x-axis.*

Example 1. *Determine if the following equations are linear:*

1. $\frac{dy}{dx} = -e^x sin(x)y + x^{2000}.?$

2. $\frac{dy}{dx} = x \cdot cos(y) + 2x.?$

METHOD: Solution of linear first-order equations

Step 1) Calculate the ________________________ factor:

Step 2) Multiply both sides of the diff. eq. by ________________________.

Step 3) Left hand side $\iff$ derivative of the product:

Step 4) Integrating both sides gives:

Step 5) Solving y, we obtain the ________________________ solution:

Example 2. *Find the general solution:*

$$(x^2+1)\frac{dy}{dx} + 3xy = 6x$$

- Is this a *linear* equation? ____________.

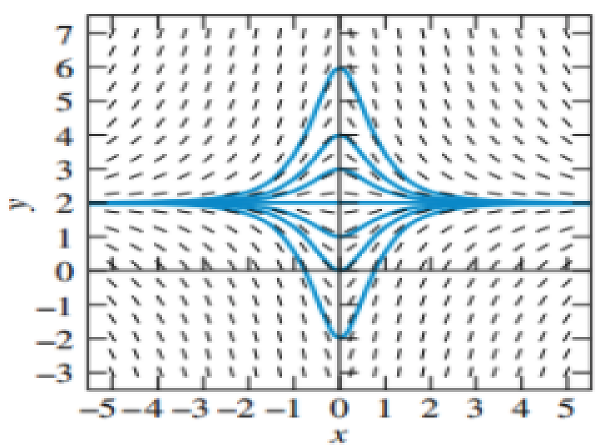
- $P(x) =$ __________________ and $Q(x) =$ __________________.

- Integrating factor:

- L.H.S. is the derivative of the product:

- Integrating both sides:

- General solution:



Example 3. *Solve the initial value problem:*

$$\frac{dy}{dx} + y = 2, \quad y(0) = 0$$

- $P(x) =$ __________________ and $Q(x) =$ __________________.
- Integrating factor:
- We know that:
- Integrating both sides:
- General solution:
- Particular solution:

Example 4. *Solve the initial value problem:*

$$x\frac{dy}{dx} + y = 3xy, \quad y(1) = 0$$

- Linear first order form:

- $P(x) =$ __________________ and $Q(x) =$ __________________.

- Integrating factor:

- We know that:

- Integrating both sides:

- General solution:

- Particular solution:

Example 5. *Find the general solution:*

$$\frac{dy}{dx} + y \cot x = \cos x$$

- $P(x) =$ __________________ and $Q(x) =$ __________________.
- Integrating factor:
- We know that:
- Integrating both sides:
- General solution:

Example 6. *Express the general solution of*

$$\frac{dy}{dx} = 1 + 2xy$$

in terms of the ***error function***

$$erf(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt.$$

- Linear first order form:

- $P(x) =$ ____________________ and $Q(x) =$ ____________________.

- Integrating factor:

- We know that:

- Integrating both sides:

- General solution: