MA 266 Lecture 7

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Sec 1.6-a Substitution Methods

- Consider the first order differential equation:

$$\frac{dy}{dx} = f(x, y) \tag{1}$$

- Suppose there exists a function:

- Suppose we can solve ____________________ for ______________:

- Then, by applying the ____________________:

- Replacing ____________________ for ______________, and solving for ______________:

- If this eq'n is *linear or separable*, then we can apply the methods from Sec. 1.4 or 1.5.

Example 1. *Solve the differential equation:*

$$x\frac{dy}{dx} = y + 2\sqrt{xy}.$$

- For ____________________, we rewrite the differential equation as:
- Let's try the substitution:
- Then
- So, the transformed equation is
- Separating variables:
- The general solution is:

Homogeneous Equations

Defintion 1. *A ________________________ first-order differential equation is one that can be written in the form:*

- If we make the substitution:

- The ________________________ is transformed into the ________________________:

- Thus every ________________________ first-order differential equation can be reduced

  to an integration problem by means of the substitutions in __________________.

Example 2. *Find general solutions of the differential equation:*

$$xy^2 \frac{dy}{dx} = x^3 + y^3$$

- For ____________, we rewrite the differential equation as:

- Substituting

- Separating variables:

- The general solution is:

Bernoulli Equation

Consider:

$$\frac{dy}{dx} + P(x)y = Q(x)y^n$$

The above equation is called a ________________________. If either __________________,

The substitution

Transforms __________________ into __________________:

Example 3. *Consider the homogeneous equation:*

$$2xy\frac{dy}{dx} = 4x^2 + 3y^2$$

- This is a *Bernoulli* equation:

- We use the substitution:

- This gives:

Example 4. *Find the general solution of the differential equation:*

$$\frac{dy}{dx} = y + y^3$$

- Rewrite the differential equation as:

- We use the substitution:

- The substitution gives:

Example 5. *The equation*

$$\frac{dy}{dx} = A(x)y^2 + B(x)y + C(x)$$

is called a ***Riccati Equation****. Suppose that one particular solution* $y_1(x)$ *of this equation is known. Show that the substitution:*

$$y = y_1 + \frac{1}{v}$$

transforms the Riccati equation into the linear equation:

$$\frac{dv}{dx} + (B + 2Ay_1)v = -A.$$

Flight Trajectories

Suppose an airplane departs from the point $(a, 0)$ located due east of its intended destination –an airport located at the origin $(0, 0)$. The plane travels with constant speed v_0 relative to the wind, which is blowing due north with constant speed w. Let's assume the plane's pilot maintains its heading directly toward the origin.

- The velocity components relative to the ground are:

- The trajectory ________________ of the plane satisfies the differential eq'n:

- Setting:

- Then ___________ takes the ______________________ form:

- Use the substitution: