MA 266 Lecture 8

Christian Moya, Ph.D.

Sec 1.6-b Exact Differential Equations

Recall that the *general* solution of

$$\frac{dy}{dx} = f(x, y) \tag{1}$$

is often defined *implicitly* by:

$$F(x, y(x)) = C. \tag{2}$$

We can recover (1) from (2) as follows:

- The general first-order differential eq'n $y' = f(x, y)$ can be written in this form with:
- As a result, if the exists a function $F(x, y)$ such that:
- ________________________ defines a general solution of __________________.
- In this case:

Theorem 1. *Suppose that the functions $M(x, y)$ and $N(x, y)$ are continuous and have continuous first-order partial derivatives in the open rectangle $R : a < x < b, c < y < d$. Then the differential equation*

$$M(x, y)dx + N(x, y)dy = 0$$

is exact in R if and only if

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x} \tag{3}$$

at each point of R. That is, there exists a function $F(x, y)$ defined on R with $\partial F/\partial x = M$ and $\partial F/\partial y = N$ if and only if (3) holds on R.

Example 1. *Verify that the following differential equation is exact; then solve it.*

$$\frac{dy}{dx} = -\frac{3x^2 + 2y^2}{4xy + 6y^2}$$

- Rewriting the equation in *differential* form gives:

- We now check if ______________ is *exact*:

Reducible Second-Order Equations

A *second-order differential equation* has the general form:

If *either* the ________________________ or the ________________________ is missing from a second-order equation, then it can be easily reduced to a first-order equation.

Dependent variable y missing.

- If y is missing, __________________ takes the form:

- Then the substitution:

- results in:

- If we can solve this equation for a general solution ________________________,

- Observe that the solution involves ____________ constants __________________.

Example 2. *Find a general solution of the differential equation:*

$$xy'' = y'$$

- Since the ______________________________ is missing, we use the substitution:

- This leads to:

- Separating variables gives:

Independent variable x missing.

- If x is missing, ___________________ takes the form:

- Then the substitution:

- results in:

- If we can solve this equation for a general solution _________________________,

- Assuming ___________________:

Example 3. *Find a general solution of the differential equation:*

$$yy'' + (y')^2 = yy'$$

- Since ________________________ is missing, we use the substitution:

- This leads to:

Example 4. *Find a general solution of the differential equation:*

$$y'' = 2y(y')^3$$

- Since _________________________ is missing, we use the substitution:

- This leads to: