MA 266 Lecture 17

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Midterm Review and Sec 3.4 Mechanical Vibrations

Example 1. *Consider a pond that initially contains 10 million gal of water. Water containing a polluted chemical flows into the pond at the rate of 6 million gal/yr, and the mixture in the pond flows out at the rate of 5 million gal/yr. The concentration* $\gamma(t)$ *of chemical in the incoming water varies as* $\gamma(t) = 2 + \sin 2t$ *grams/gal. Let* $Q(t)$ *be the amount of chemical at time* t *measured by millions of grams. Derive the differential equation of the process.*

Example 2. *Let $y(t)$ be the solution of the IVP:*

$$y''' + y' = 0, \quad y(0) = 2, \quad y'(0) = 1, \quad y''(0) = 1,$$

then $y(\pi) =$?

Example 3. *Find the particular solution of the IVP:*

$$y' = \frac{1-2x}{y}, \quad y(1) = -2,$$

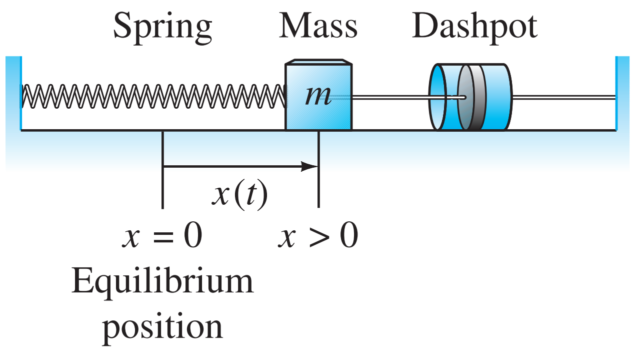
in explicit form.

Example 4. *Find the solution of the IVP*

$$y'' + y' - 6y = 0, \quad y(0) = 0,\ y'(0) = 5.$$

Mechanical Vibrations

In Sec 3.1, we considered the following mass connected to a spring and a dashpot.



We described the dynamics of this system using the linear equation:

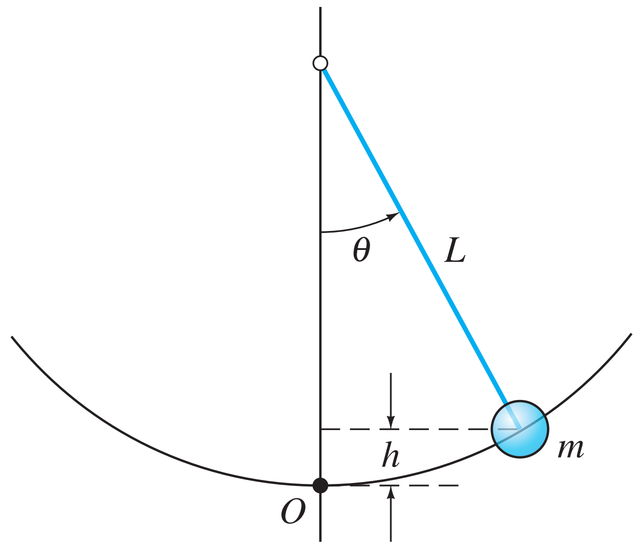
Here

- ____________________: spring force, ____________________: spring constant
- ____________________: dashpot force, ____________________: damping constant
- ____________________: external force.

Remarks

- ____________________: no dashpot $\iff$
- ____________________: if ______________.
- Motion is ____________________ if ____________________.
- Motion is ____________________ if ____________________.

The Simple Pendulum



We let

- ____________________: arc distance from O to m.

- ____________________: velocity of m

Kinetic Energy:

Potential Energy:

The sum of the kinetic energy T and potential energy V:

Differentiating with respect to t both sides:

Note: We can also obtain the above differential equation using Newton's second law.

Going from nonlinear to linear

- Small angle approximation:

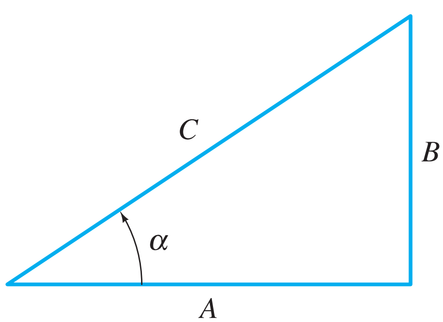
- Adding *frictional resistance:* ____________________

FREE Undamped Motion

- Define:

- General solution:

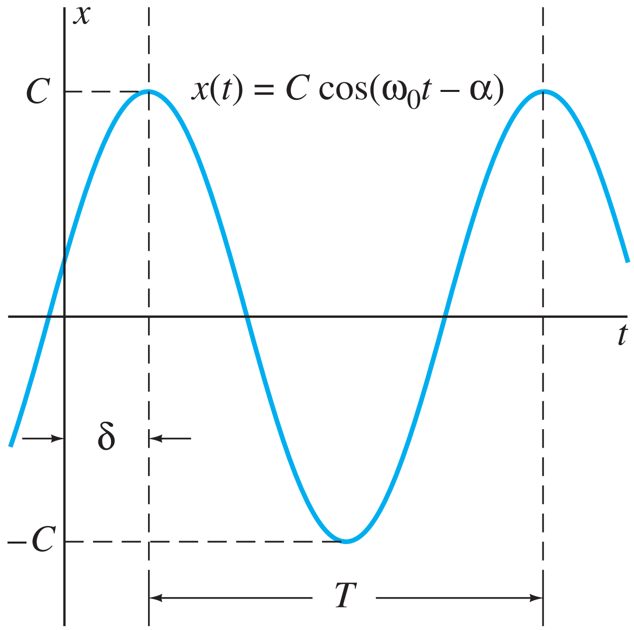
Phase angle



- General solution:

The mass oscillates with:

- Amplitude: ____________________.
- Circular frequency: ____________________.
- Phase angle: ____________________.



- Period:

- Frequency:

- Time lag:

FREE Damped Motion

- Characteristic equation:

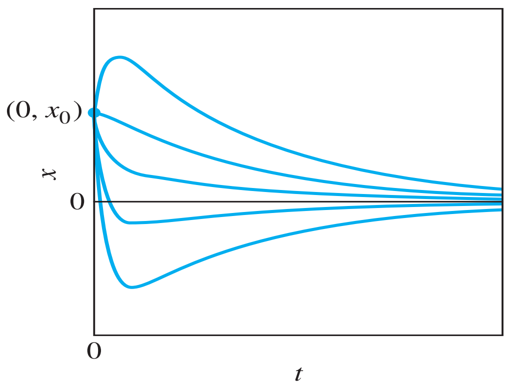
- Sign depends on:

- **Critical damping:**

Overdamped case $c > c_r$

- $\implies$ two *distinct* roots:

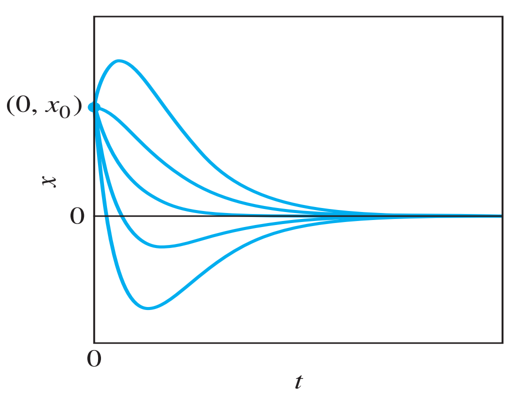
- General solution:



Critically damped case $c = c_r$

- $\implies$ *repeated* roots:

- General solution:

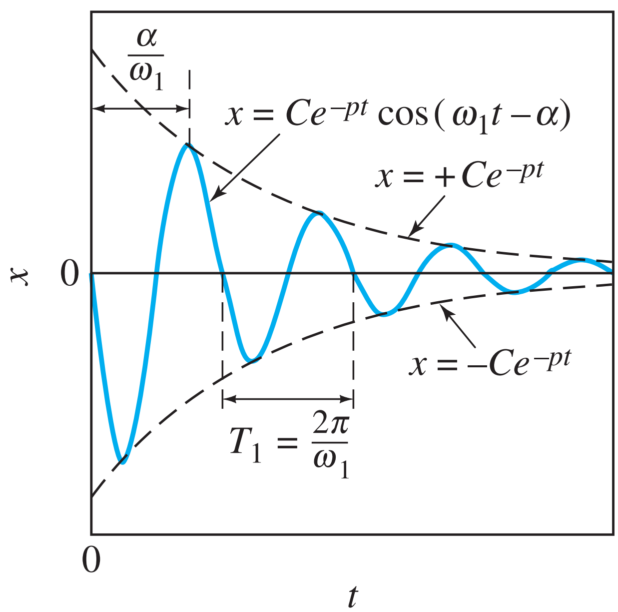


Underdamped case $c < c_r$

- $\implies$ two *complex* roots:

- General solution:

- Using derivation ____________:



Example 5. *Consider the differential equation of a spring-mass-(dashpot) system:*

$$mx'' + cx' + kx = 0.$$

Find the particular solution

a) *with damping:* $m = 1, c = 10, k = 125, x_0 = 6,$ *and* $v_0 = 50$.

b) *without damping:* $m = 1, k = 125, x_0 = 6,$ *and* $v_0 = 50$.

Solution a):

- Characteristic equation:

- General solution:

- Particular solution:

Solution b):

- Characteristic equation:

- General solution:

- Particular solution: