MA 266 Lecture 22

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Practical Resonance - Forced Damped Oscillations

- Consider the damped system:

$$mx'' + cx' + kx = F_0 \cos \omega t$$

- Note that if $c > 0$, then the "forced amplitude" $C(\omega)$:

$$C(\omega) = \frac{F_0}{\sqrt{(k - m\omega^2)^2 + (c\omega)^2}} < +\infty,$$

  always remains *finite*.

- However, the forced amplitude may attain a maximum for some value of ω, in which case we speak of *practical resonance*.

Example 1. *Show that if $c \geq \sqrt{2km}$ the amplitude $C(\omega)$ decreases for all $\omega > 0$; otherwise $C(\omega)$ attains a maximum value.*

1. Use $C'(\omega)$:

2. If $c \geq \sqrt{2km}$:

3. But if $c < \sqrt{2km}$:

Example 2. *Find the amplitude $C(\omega)$ and the practical resonance frequency ω of the following forced mass-spring-dashpot system:*

$$x'' + 10x' + 650 = 100\cos\omega t.$$

- Particular solution:

- Solving for the coefficients A and B

- The amplitude $C(\omega)$ of the steady periodic forced oscillations with freq. ω :

- Find the practical resonance by solving $C'(\omega) = 0$:

4.1 First-Order Systems

- We have discussed methods for solving an ordinary differential equation that involves only one dependent variable.
- Many applications, however, require the use of two or more dependent variables, each a function of a single independent variable (typically time).
- Such a problem leads naturally to a *system* of simultaneous ODEs.

Notation:

- We will usually denote the independent variable by ____________ and the dependent variables (the unknown functions of t) by:

- We will restrict our attention to systems in which the number of equations is the same as the number of dependent variables (unknown functions).

Solutions:

- For instance, a system of two first-order equations in the dependent variables x and y has the general form

  where the functions f and g are given.

- A *solution* of this system is a pair________________________ of functions of t that satisfy both equations identically over some interval of values of t.

First-Order Systems

- Consider a system of differential equations that can be solved for the highest-order derivatives of the dependent variables.

- For instance, in the case of a system of two second-order equations:

- These can be transformed into an equivalent system of *first-order* equations.

First-Order Systems

- Consider first the "system" consisting of the single n th-order equation

- We introduce the dependent variables x_1, x_2, $\ldots$, x_n defined as follows:

- Note that

  and so on.

Equivalent System

- Substitution then yields the system

  of n *first-order* equations.

Example 3. *Transform the following differential equation into an equivalent system of first-order differential equations:*

$$x'' + 4x - x^3 = 0.$$

- The above equation is of the form:

- Substitution:

- yields the system:

Example 4. *Transform the following system of differential equations into an equivalent system of first-order differential equations:*

$$\begin{aligned} 2x'' &= -6x + 2y \\ y'' &= 2x - 2y + 50 \sin 5t. \end{aligned}$$

- Substitution:

- yields the system:

Example 5. *Transform the following differential equation into an equivalent system of first-order differential equations:*

$$x^(3) - 2x'' + x' = 1 + te^t.$$

- Substitution:

- yields the system:

Example 6. *Transform the following system of differential equations into an equivalent system of first-order differential equations:*

$$\begin{aligned} x'' &= (1 - y)x \\ y'' &= (1 - x)y. \end{aligned}$$

- Substitution:

- yields the system:

Example 7. *Solve the following IVP:*

$$x' = -2y,\ y' = 2x;\ x(0) = 1,\ y(0) = 0.$$