MA 266 Lecture 23

Christian Moya, Ph.D.

Sec 4.1 First-Order Systems

Example 1. *Solve the following IVP:*

$$x' = -y, \; y' = 13x + 4y; \; x(0) = 0, \; y(0) = 3.$$

- The computation:

- yields

- Characteristic equation:

- General solution:

- Then the original first equation $y = -x'$ gives:

- We find the *particular solution* using the ICs:

Sec 4.2 The Method of Elimination

- A **linear** first order system is of the form:

- We say that the system is *homogeneous* if ________________________.
- Otherwise, it is *nonhomogeneous*.

The Method of Elimination

- The *method of elimination* for linear differential systems is *similar* to the solution of a linear system of algebraic equations.
- The *method of elimination* corresponds to the process of *eliminating* the unknowns one at a time until only *a single* equation with *a single* unknown remains.

Example 2. *Find the particular solution of the following linear system (IVP) using the method of elimination:*

$$x' = 3x - y, y' = 5x - 3y; x(0) = 1, y(0) = -1.$$

- Substitution of

- yields

- General solution:

- Substitution of this solution in

- Using the ICs $x(0) = 1, y(0) = -1$, we have

- Hence the desired particular solution is

Example 3. *Find the general solution of the following linear system:*

$$x' = 2x + y, y' = x + 2y - e^{2t}.$$

- Substitution of

- yields

- Trial solution:

- Coefficients:

- Hence the general solution is:

- Substitution in

Example 4. *Find the general solution of the following linear system:*

$$x'' = 6x + 2y, y'' = 3x + 7y.$$