# MA 266 Lecture 31

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## Sec 7.1 Laplace Transforms and Inverse Transforms

- **Idea:** The transforms ____________ and ____________ are ____________ and ____________.

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**Definition -** Laplace Transform

- Given a function $f(t)$ defined for all $t \geq 0$, the *Laplace transform* of $f$ is the function $F$ defined as follows:

  for all values of $s$ for which the improper integral converges.

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- The *improper integral* over an infinite interval is defined as

- If the limit exists, we say the integral

  1. ________________________________

  2. ________________________________

**Example 1.** *Use the previous definition to find the Laplace transform of*

$$f(t) = 1.$$

**Solution.**

- For $t \geq 0$, applying the definition gives

- if

- So, it follows that

**Example 2.** *Use the previous definition to find the Laplace transform of*

$$f(t) = e^{at}.$$

**Solution.**

- For $t \geq 0$, applying the definition gives

- if

- So, it follows that

**Example 3.** *Use the definition to find the Laplace transform of*

$$f(t) = t^a.$$

*where $a$ is real and $a > -1$.*

**Solution.**

The Laplace transform $\mathcal{L}\{t^a\}$ of a power function is most conveniently expressed in terms of the **gamma function** $\Gamma(x)$, which is defined for $x > 0$ by the formula

If $n$ is a positive integer:

- if we substitute

**Theorem** - Linearity of transform If $a$ and $b$ are constants, then

for all $s$ such that the Laplace transforms of the functions $f$ and $g$ both exists.

**Proof.**

**Example 4.** *Find the Laplace transform of* $f(t) = \sin kt$.

- Recall that

- Then, it follows from the linearity of the Laplace transform that

**Definition** - *Piecewise Continuous Functions*

The function $f(t)$ is said to be **piecewise continuous** on the bounded interval $[a, b]$ provided that $[a, b]$ can be subdivided into finitely many subintervals in such a way that

1. $f$ is continuous in the interior of each of the subintervals
2. $f(t)$ has a finite limit as $t$ approaches each endpoint of each subinterval from its interior.

For *e.g.,*

**Example 5.** *Find the Laplace transform of the above piecewise continuous function.*

- We begin with the definition of the Laplace transform. We obtain

- Consequently

**Example 6.** *Find the Laplace transform of* $f(t) = te^t$.

**Solution.**

**Definition** - *Inverse transform*

If $F(s)$ is the transform of some continuous function $f(t)$, then $f(t)$ is uniquely determined. Thus, if $F(s) = \mathcal{L}\{f(t)\}$, then we call the **inverse Laplace transform** of $F(s)$ and write

**Example 7.** *Find the inverse Laplace transform of*

$$F(s) = \frac{1}{s+5}.$$

**Solution.**

**Example 8.** *Find the inverse Laplace transform of*

$$F(s) = s^{-3/2}.$$

**Solution.**