Reading Course

Math 598: Introduction to Functional Analysis

Fall 2007

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Meeting times: Arranged

BOOKS

TEXT: The official texts for the course are *A Course of Functional Analysis* by John B. Conway, *Introduction to Functional Analysis* by Angus Taylor and David Lay, and *Introduction to Banach Space Theory* by Robert Megginson. You will also be given some "lecture notes" of the instructor which will follow the spirits of the books but not necessarily the order of topics of the books. You will *not* be required to purchase any books as they may be consulted in the library and no assignments will be made from the books.

RESERVE: The following books will be on Reserve in the Mathematics Library.

J. B. Conway	*A Course of Functional Analysis*
R. E. Megginson	*Introduction to Banach Space Theory*
A. Taylor and D. Lay	*Introduction to Functional Analysis*
P. R. Halmos	*Introduction to Hilbert Space*
P. R. Halmos	*Hilbert Space Problem Book*
W. Rudin	*Real and Complex Analysis*
W. Rudin	*Functional Analysis*

NON-RESERVE: The following books are not on reserve for this course, but might be useful references. Except for Baggett and Groetsch, they are more advanced than our text.

L. W. Baggett	*Functional Analysis: A Primer*
S. K. Berberian	*Lectures in Functional Analysis*
C. W. Groetsch	*Elements of Applicable Functional Analysis*
Dunford and Schwartz	*Linear Operators* (3 Volumes)
Riesz and Sz. Nagy	*Functional Analysis*
K. Yosida	*Functional Analysis*

TOPICS

- Definitions, examples, and elementary properties
- Linear functionals and convexity: extension and separation theorems, weak topologies, Krein-Milman theorem
- Duality
- Completeness: uniform boundedness, open mapping, and closed graph theorems
- Hilbert spaces and generalized Fourier series
- Operators: spectrum, spectral mapping theorem, Riesz functional calculus
- Compact Operators
- Spectral theorem for self-adjoint and normal operators on Hilbert space
- Unbounded operators (if time permits)

GRADING POLICY AND PHILOSOPHY

3 Keys to Learning Mathematics

1. Work lots of problems.
2. Memorize the definitions and the statements of the major theorems.
3. Work lots more problems.

This course is not at the research level, but is just below that level. At this time, you should be learning to do and write mathematical research. Also, this course is frequently included in the syllabus for Advanced Topics Examinations. The grading policy has been chosen to reinforce these goals.

Ordinarily, after mathematicians complete research on a topic, they write up their results and submit them for publication to an appropriate journal. To be accepted for publication, the work must be interesting, correct, and reasonably well written. For the purposes of this class, all homework problems are, by hypothesis, interesting. After you submit them, they will be accepted if they are correct and reasonably well written. (Note that neither mathematics journals, your thesis committee, nor this class will accept work that is half correct.)

Publication of jointly authored research in mathematics is frequent and acceptable: the authors do the research together and write up a single account of their work for submission with all their names as authors.

There is an unwritten code of ethics governing joint publication. Whether work discussed with another mathematician is "joint" or not is determined by the individuals involved by mutual consent. One standard is that the work is jointly authored if the contribution of each author is "significant". It is unethical to submit jointly authored work as your own. It is equally unethical to include an author who has contributed nothing. Assignment of credit by the mathematical community for jointly authored work is always problematic and ranges from the generous: each author gets full credit for the whole paper; to the blantantly unfair: you get no credit, the other author gets full credit.

Joint authorship of homework will be acceptable in this class (to encourage discussion of the problems among you) and the usual ethical standards and the usual procedures apply: one manuscript with all authors names will be submitted for grading. Credit will be given for a problem only after the problem has been accepted as correct and reasonably well written. For a paper with $n > 1$ authors, credit to each author will be $2/n$, so, for example, for an individually worked problem, the author will get 1 credit; for a problem with two authors, *each* author will receive $2/2 = 1$ credit; whereas for a problem with three authors, *each* author will receive $2/3$ credit.

> Grades will be based on submitted homework (20%),
> accepted homework (60%), and
> on a written final examination given during finals week(20%).

HOMEWORK: You will be given a list of about 100 problems; you are encouraged to do as many of these as possible. The problems are of uneven difficulty: some are very easy; some of the more difficult ones are starred (*). You may turn in the problems in any order, and at any time, except that half the submitted problems must be on material from the first half of the course (on spaces, roughly speaking) and half on material from the second half of the course (on operators, roughly speaking).

- You should submit *at least* 30 problems during the semester:
  at least 4 by August 31,
  at least 4 more by September 14,
  at least 4 more by September 28,
  at least 4 more by October 12,
  at least 4 more by October 26,
  at least 4 more by November 9,
  at least 4 more by November 30,
  No solutions or corrections accepted after noon, Friday, December 14.

  Grades for the submission of homework will be the percentage of these goals: for example, 3 problems instead of 4 on each of the dates will result in a grade of 75% (maximum, 100%).

- Rejected problems may *(should)* be resubmitted after correction.

- Grades for accepted homework will be computed as follows:

problem credits < 15	C
$15 \le$ problem credits < 30	B
$30 \le$ problem credits < 45	A
$45 \le$ problem credits	A and *the final is waived!*

FINAL EXAM: The Final Exam will consist entirely of questions concerning definitions and theorems from the course. There will be no "problems" on the final exam. The goal of the final exam is to encourage you to learn the facts of the course.

NOTE: I expect everyone to get an "A" for the course!