Problem set 1- due September 9. Write clear and complete solutions for each problem. You may send them to the grader Gavin Glenn (email: glenng at purdue dot edu) and cc me (preferred method), or hand it to me in class. Please make your writing clearly legible.

1a) Denote by $\mathbb{R}$ the set of real numbers. An *open interval* is a subset $D \subset \mathbb{R}$ such that there exists $a, b \in \mathbb{R}$, $a < b$, and

$$D = (a, b) = \{y \in \mathbb{R} | a < y < b\}.$$

In a one-variable calculus course continuity is usually defined as follows:

Let D be an open interval. A function $f : D \to \mathbb{R}$ is said to be *continuous at* $c \in D$ if for any given real number $\epsilon > 0$ there exists a real number $\delta > 0$ such that $x \in D$ and $|x - c| < \delta$ implies $|f(x) - f(c)| < \epsilon$. We say f is *continuous* if it is continuous at all $c \in D$.

If D is an open interval and $f : D \to \mathbb{R}$ a function, prove the following are equivalent:

i) f is continuous at $c \in D$

ii) For any open interval $(a, b) \subset \mathbb{R}$ containing $f(c)$ there exists another open interval $(a', b') \subset D$ containing c such that $f((a', b')) \subset (a, b)$.

iii) For any open interval $(a, b) \subset \mathbb{R}$ containing $f(c)$, there exists some open interval $(a', b') \subset f^{-1}((a, b))$ containing c.

1b) Let $\mathcal{T}$ be the standard topology on $\mathbb{R}$. Prove that $f : D \to \mathbb{R}$ is continuous if and only if for any open set $U \in \mathcal{T}$ we have $f^{-1}(U) \in \mathcal{T}$.

2a) Suppose X is a set and $\{\mathcal{T}_\alpha\}_{\alpha \in J}$ is a collection of topologies on X indexed by some set J. Is the intersection

$$\bigcap_{\alpha \in J} \mathcal{T}_\alpha$$

a topology on X? If yes, explain with a proof, if not give a counterexample.

2b) Suppose $\mathcal{T}$ and $\mathcal{T}'$ are two topologies on a set X. Is the union $\mathcal{T} \cup \mathcal{T}'$ a topology on X? If yes, explain with a proof, if not give a counterexample.

Exercises from Munkres: 13.1, 13.3, 13.8, 16.1, 16.3, 16.4, 16.6, 16.9, 16.10.