

Problem set 2- due September 23. Write clear and complete solutions for each problem. You may send them to the grader Gavin Glenn (email: glenn.g at purdue dot edu) and cc me (preferred method), or hand it to me in class. Please make your writing clearly legible.

1) For this problem consider the set of complex numbers \mathbb{C} equipped with the standard topology (through the identification with \mathbb{R}^2 by sending $a + bi$ to (a, b)) and $\mathbb{C} \times \mathbb{C}$ with the product topology. Prove that

a) the complex conjugation map $\mathbb{C} \rightarrow \mathbb{C}$, defined by sending $a + bi$ to $a - bi$, is continuous.

b) the complex multiplication map

$$\mu : \mathbb{C} \times \mathbb{C} \rightarrow \mathbb{C}$$

$$\mu(a + ib, c + id) = (ac - bd) + i(ad + bc)$$

is continuous.

2) A topological space X is said to be **connected** if the only subsets of X that are simultaneously closed and open are the empty set and X itself.

a) Prove that X is connected if and only if the only continuous functions from X to $\{0, 1\}$ (with the discrete topology) are the constant functions.

b) Suppose X and Y are topological spaces and $f : X \rightarrow Y$ a surjective continuous function. Prove that if X is connected, then Y is also connected.

Exercises from Munkres: 17.8 – 17.13, 17.19, 18.3, 18.5, 18.8, 18.9, 18.13