

Problem set 5- due November 4. Write clear and complete solutions for each problem. You may send them to the grader Gavin Glenn (email: glenn.g at purdue dot edu) and cc me (preferred method), or hand it to me in class. Please make your writing clearly legible.

1) Show \mathbb{R} is homeomorphic to $(0, +\infty)$. Show \mathbb{R} is not homeomorphic to $[0, +\infty)$.

Hint: For the latter, see what happens if you remove a point from each of these spaces.

2) Let $S^1 = \{x \times y \in \mathbb{R}^2 \mid x^2 + y^2 = 1\}$ be the circle with subspace topology.

a) Prove S^1 is not homeomorphic to an open interval.

b) Prove S^1 is not homeomorphic to a closed interval.

3) Use the intermediate value theorem to show that if $f : S^1 \rightarrow \mathbb{R}$ is a continuous map then there exists a point $x \in S^1$ such that $f(x) = f(-x)$. In other words, there will be always two antipodal points in the equator of the earth with the same temperature :)

Exercise 23.3, 23.5, 23.6, 23.7