Problem set 2- due September 30. Write clear and complete solutions for each problem. You may send them to me via email (cui177 at purdue dot edu, preferred method) or in class. Please make your writing clearly legible.

1. Let G be a group, and $w \in Z^3(G;\mathbb{C}^\times)$ be a normalized 3-cocycle of G with coefficient $\mathbb{C}^\times = \mathbb{C} - \{0\}$. Recall the monoidal category $\mathcal{C}_G^w$ where the objects are elements of G, tensor product is given by group multiplication in G, and w is the associator $w(g,h,k) : (g \otimes h) \otimes k \to g \otimes (h \otimes k)$. Here, w is normalized, i.e., $w(g,e,h) = 1$ where e is the identity element in G.

   - Show explicitly that $w(g,h,k) = 1$ whenever at least one of g, h, k is e. (Note that this is implied by Mac Lane's coherence theorem, but here you're asked to prove it directly using the normalized cocycle condition.)
   - For two normalized cocycles w, w', show that if they are cohomologous, then $\mathcal{C}_G^w$ and $\mathcal{C}_G^{w'}$ are monoidally equivalent.
   - For each object g, show that g^{-1} is both a left and right dual of g with the following morphisms:

   $$b_g := id_e : e \to g \otimes g^{-1}, \qquad d_g := w(g,g^{-1},g)^{-1}\ id_e : g^{-1} \otimes g \to e$$

   $$b'_g := id_e : e \to g^{-1} \otimes g, \qquad d'_g := w(g,g^{-1},g)\ \ id_e : g \otimes g^{-1} \to e$$

   Hence $\mathcal{C}_G^w$ is rigid monoidal category. (You can defined the $b's$ and $d's$ differently from above, but they are unique up to isomorphism; recall the uniqueness of left/right duals.)

2. Let $\mathcal{C}$ be a monoidal category with $\mathbf{1}$ the tensor unit. Show that $\mathrm{End}(\mathbf{1}) := \mathrm{Hom}(\mathbf{1},\mathbf{1})$ is commutative monoid. You can use the fact that $l_{\mathbf{1}} = r_{\mathbf{1}} : \mathbf{1} \otimes \mathbf{1} \to \mathbf{1}$.