Problem set 3- due October 14. Write clear and complete solutions for each problem. You may send them to me via email (cui177 at purdue dot edu, preferred method) or in class. Please make your writing clearly legible.

1. A *symmetric* monoidal category is a braided monoidal category such that the braiding satisfies $c_{W,V} \circ c_{V,W} = id$, for all objects V and W. Let $\mathcal{C}$ be a symmetric monoidal category with the left duals.

   - Show that $\mathcal{C}$ is a ribbon category by defining the twist θ to be the identity natural isomorphism.
   - Show that the (framed oriented) link invariant associated with $\mathcal{C}$ is not interesting in that the invariant of any $\mathcal{C}$-colored link is equal to that of the unlink with the same number of components and the same color.
   - For any group G, show that Rep(G), the category of finite dimensional representations of G over a field, is a symmetric monoidal category. (You can assume Rep(G) is a monoidal category.)

2. Let A be an Abelian group, and $b : A \times A \to \mathcal{C}^\times$ a bi-character where $\mathcal{C}^\times$ is the multiplicative group of non-zero complex numbers. Consider the monoidal category $\mathcal{C}_G$, a strict monoidal category with objects given by elements of G. The bi-character b defines a braiding on G by setting the braiding $c_{g,h} := b(g, h)$.

   - Extend the braided category obtained above to a ribbon category.
   - Let A be the cyclic group of order $N > 2$. Give a bicharacter on A such that the resulting ribbon category is not symmetric.

3. Let $\mathcal{C}$ be a strict monoidal category with left duals. For any objects V, W, show that $(V \otimes W)^* \cong W^* \otimes V^*$. Hint: thanks to the uniqueness of duals, you can try to define the morphisms $b_{V \otimes W} : \mathbf{1} \to (V \otimes W) \otimes (W^* \otimes V^*)$ and similarly $d_{V \otimes W}$ such that they satisfy the rigidity conditions. Picture calculus could be useful.