**MA262 — REVIEW EXERCISES FOR EXAM I-
BASED ON OLD MA262 EXAMS WHICH CAN BE FOUND AT**

https://www.math.purdue.edu/academic/courses/oldexams.php?course=MA26200

In case you want to do other problems from old exams, be aware that we have only been using the current textbook since the fall 2020 and some problems of exams given before fall 2020 may not match the current topics covered in this course. The notation may also be quite different.

Because problems are taken from old test pages which may have page numbers, “actual” page numbers for this document are shown in red.

1. Let $y(x)$ satisfy

$$y' + \frac{4}{x}y = 12x, \\ y(1) = 8.$$

Then $y(2)$ is equal to

A. $\frac{25}{4}$

B. $\frac{32}{9}$

C. $\frac{67}{8}$

D. $\frac{23}{2}$

E. $\frac{47}{8}$

2. Let $y(x)$ be the solution of

$$(y^2 + 2x + \cos x)dx + (2xy + \sin y)dy = 0, \\ y(0) = \pi.$$

If we denote $y(\frac{\pi}{2}) = Y$, we can say that Y satisfies the following equation:

A. $\frac{\pi}{2}Y^2 + \frac{\pi^2}{4} - \cos Y = 0.$

B. $\frac{\pi}{2}Y^3 + \frac{\pi^2}{4} - \cos Y = 1.$

C. $\frac{\pi}{2}Y^2 + \frac{\pi^2}{4} + \sin Y = 0.$

D. $\frac{\pi}{2}Y^2 - \frac{\pi^2}{4} + \sin Y = 0.$

E. $\frac{\pi}{4}Y^2 + \frac{\pi^2}{4} - (\cos Y)^2 = 0.$

3. Let $y(x)$ satisfy

$$y'(x) + \frac{2x}{1+x^2}y = -2xy^2,$$
$$y(0) = 1.$$

Then $y(1)$ is equal to

A. $y(1) = \dfrac{1}{2+2\ln 2}$

B. $y(1) = \dfrac{1}{1+\ln 2}$

C. $y(1) = 2(\ln 2) + 1$

D. $y(1) = 2 + (\ln 2)$

E. $y(1) = \dfrac{1}{3+\ln 2}$

4. The solution of the differential equation

$$\frac{dy}{dx} = \frac{x^3+4y^3}{3xy^2}, \quad x > 0$$

is defined implicitly by the following equation:

A. $x^3 + y^3 - Cxy^3 = 0$

B. $(x+y)^3 - Cx = 0$

C. $x^3 + y^3 - Cx^4 = 0$

D. $x + y - Cx^2 = 0$

E. $1 + y^3 - Cx = 0$

5. Let $y(x)$ satisfy the following second order differential equation

$$yy'' = 3(y')^2,$$
$$y(0) = 1, \quad y'(0) = 1.$$

Then $y(\frac{3}{8})$ is equal to

A. $y(\dfrac{3}{8}) = 1$

B. $y(\dfrac{3}{8}) = 2$

C. $y(\dfrac{3}{8}) = 3$

D. $y(\dfrac{3}{8}) = 4$

E. $y(\dfrac{3}{8}) = 5$

6. Find the solution of the initial value problem

$$xy' = y + \frac{5}{4}(x^4 y)^{\frac{1}{5}},$$
$$y(1) = 1.$$

A. $y(x) = x(1 + \ln x)^{\frac{5}{4}}$

B. $y(x) = x^2(1 + \ln x)^{\frac{3}{4}}$

C. $y(x) = x(1 + x\ln x)^{\frac{1}{4}}$

D. $y(x) = x(1 + 3\ln x)^{\frac{5}{4}}$

E. $y(x) = x(1 + 5\ln x)^{\frac{5}{4}}$

7. An object with initial temperature 32F is placed in a refrigerator whose temperature is a constant 0F. An hour later the temperature of the object is 16F. What will its temperature be four hours after it is placed in the refrigerator? Hint: Newton's law of cooling $\frac{dT}{dt} = -k(T - T_m)$.

A. 1

B. 2

C. 3

D. 4

E. 5

8. The population of a certain species obeys the equation

$$\frac{dp}{dt} = 10(200 - p)(p - 300).$$

If the initial population is 100, what will the approximate value of the population be after a long time?

A. 100

B. 200

C. 300

D. 150

E. The population will be extinct .

9. A tank originally contains 100 gal of water with a salt concentration of $\frac{1}{2}$ lb/gal. A solution containing a salt concentration of 2 lb/gal enters at a rate of 2 gal/min and the well-stirred mixture is pumped out at the rate of 1 gal/min. The amount of salt in the tank after 50 min is

A. 0 lb

B. $400 - 350e^{\frac{1}{2}}$ lb

C. e^{-2} lb

D. 100 lb

E. 200 lb

10. Consider the system

$$\begin{aligned} x_1 + x_2 + x_3 &= 2 \\ 2x_1 + 3x_2 + 2x_3 &= 5 \\ 2x_1 + 3x_2 + (k^2 - 2)x_3 &= k + 7 \end{aligned}$$

Determine all the values of the constant k for which the above system has no solutions.

A. $k = -2$

B. $k = 2$

C. $k \neq -2$

D. $k \neq 2$

E. $k = 3$

5. If the system

$$\begin{aligned} x - y - 2z &= a \\ 2x + 2y - 3z &= b \\ 3x + y - 5z &= c \end{aligned}$$

is consistent, what can we conclude about a, b, and c?

A. $c = a$

B. $c = a - b$

C. $c = a + b$

D. $c = -a - b$

E. a, b, and c can be any numbers.

17. Solve the differential equation

$$(2xy + x^3)dx + (x^2 + y^3 + 2)dy = 0, \quad y(0) = 2.$$

A. $x^2y + 2y = 4$

B. $x^4 + 2y = 8$

C. $x^2y + \frac{1}{4}x^4 + \frac{1}{4}y^4 + 2y = 8$

D. $x^2y + \frac{1}{4}x^4 + \frac{1}{4}y^4 = 0$

E. $\frac{1}{4}x^4 + \frac{1}{4}y^4 = 8$

1. If $xy' - 3y = x^3$ and $y(1) = 1$, then $y(e) =$

A. e^4
B. e^{-2}
C. $2e^{-3}$
D. $2e^3$
E. e^3

2. The general solution of

$$(2x^2y)y' = -3x^2 - 2xy^2$$

is:

A. $x^2y^3 + y^3 = C$
B. $x^2y^2 + x^3 = C$
C. $x^2y^2 = C$
D. $x^3y^2 + x^2 = C$
E. $x^2y^3 + x = C$

2. If the system

$$\begin{aligned} 3x + y - 5z &= a \\ 2x + 2y - 3z &= b \\ x - y - 2z &= c \end{aligned}$$

is consistent, what can we conclude about a, b and c?

A. $c^2 = a^2$

B. $a + b = 6$

C. $c = 3$

D. $c = a - b$

E. $c = a + b$

3. The general solution of $xy' - y = x^2e^x$ is

A. $y = xe^x + cx$

B. $y = x^2e^x - xe^x + cx$

C. $y = xe^x - cx^2$

D. $y = x^2e^x + xe^x + cx$

E. None of the above

4. The solution of $(3x^2 + y)dx + (x + 2y)dy = 0$ passing through the point $(1, 1)$ is

A. $x^2 + xy + y^2 = 3$

B. $x^2 + xy + y^3 = 3$

C. $x^2 + x + y^2 = 3$

D. $x^3 + xy + y^2 = 3$

E. $x^3 + x^2y + y^3 = 3$

2. If y is the solution of the initial value problem

$$\frac{dy}{dx} = \frac{2x(y-2)}{x^2+1}, \qquad y(0) = 4,$$

then $y(1) =$

A. 4

B. 6

C. 8

D. 10

E. 12

2. (8 points) For $x > 0$, the solution to the equation

$$\left(\frac{2y}{x} + 2x\right) + (2\ln(x) - 3)\,y' = 0$$

is given implicitly by

A. $y(2\ \ln(x) - 3) + x^2 = c$

B. $y(2\ \ln(x) + 3) - 3x^2 = c$

C. $\ln(x) - x^3 - 3y + c = 0$

D. $y(\frac{2}{x} + \frac{3}{2}) + x^2 + c = 0$

E. $\frac{1}{x} + x^2 - y^2 + c = 0$

10. For which value of c does the following system have infinitely many solutions?

$$\begin{cases} 3x - 2y + 5z = 1 \\ 2y + z = 1 \\ -3x + 6y + cz = 1 \end{cases}$$

A. -3

B. -1

C. $1/2$

D. $5/2$

E. 2

10. For a real number a, consider the system of equations

$$\begin{array}{rcrcrcr} x & + & y & + & z & = & 2 \\ 2x & + & 3y & + & 8z & = & 4 \\ 2x & + & 3y & + & (a^2-1)z & = & a+1 \end{array}$$

Which of the following statements is true?

A. If $a = 0$ the system is inconsistent.

B. If $a = 2$ the system has infinitely many solutions.

C. If $a = -2$ then the system has at least two distinct solutions.

D. If $a = 3$ then the system has a unique solution.

E. If $a = -3$ then the system is inconsistent.

8. (8 points) Consider the linear system

$$\begin{aligned} x_1 + x_3 &= 2 \\ x_1 + 2x_2 - x_3 &= 4 \\ 3x_1 + x_2 + 2x_3 &= 7 \end{aligned}$$

Which of the following gives the solutions of the system in parametric form?

A. $\mathbf{x}=\begin{bmatrix}2\\1\\0\end{bmatrix}+\text{t}\begin{bmatrix}-1\\1\\1\end{bmatrix}$ with any $t \in \mathbb{R}$

B. $\mathbf{x}=\begin{bmatrix}2\\1\\0\end{bmatrix}+\text{t}\begin{bmatrix}1\\-1\\1\end{bmatrix}$ with any $t \in \mathbb{R}$

C. $\mathbf{x}=\begin{bmatrix}2\\1\\0\end{bmatrix}+\text{t}\begin{bmatrix}1\\1\\1\end{bmatrix}$ with any $t \in \mathbb{R}$

D. $\mathbf{x}=\text{s}\begin{bmatrix}2\\1\\0\end{bmatrix}+\text{t}\begin{bmatrix}-1\\1\\1\end{bmatrix}$ with any $s, t \in \mathbb{R}$

E. $\mathbf{x}=\text{s}\begin{bmatrix}2\\1\\0\end{bmatrix}+\text{t}\begin{bmatrix}1\\-1\\1\end{bmatrix}$ with any $s, t \in \mathbb{R}$

7. The graph of a solution of the differential equation

$$(x^2y + x)dy + (xy^2 + y)dx = 0$$

passes through the point $(x, y) = (1, -1)$. If it also pass through the point $(x, y) = (2, a)$, then find a.

A. $a = 2$

B. $a = -\frac{1}{2}$

C. $a = \sqrt{2}$

D. $a = -\sqrt{2}$

E. $a = -1$

8. Classify the equilibrium solutions for the following differential equation,

$$y' = y^2(y^2 - 1).$$

A. $y(t) = -1$ is stable ; $y(t) = 0$ is semi-stable and $y(t) = 1$ is unstable

B. $y(t) = -1$ is unstable ; $y(t) = 0$ is semi-stable and $y(t) = 1$ is stable

C. $y(t) = -1$ is semi-stable ; $y(t) = 0$ is semi-stable and $y(t) = 1$ is stable

D. $y(t) = -1$ is stable ; $y(t) = 0$ is unstable and $y(t) = 1$ is stable

E. $y(t) = -1$ is stable ; $y(t) = 0$ is stable and $y(t) = 1$ is stable

6. Let $y(x)$ be the solution of the initial value problem

$$y' + y = y^2 e^x,$$
$$y(0) = 1.$$

Find $y(\frac{1}{2})$.

A. $-2e^{-\frac{1}{2}}$

B. $2e^{-\frac{1}{2}}$

C. $e^{-\frac{1}{2}}$

D. $-3e^{-\frac{1}{2}}$

E. $3e^{-\frac{1}{2}}$

4. Let $y(x)$ be the solution of the following initial value problem

$$y'' + 2y^{-1}(y')^2 = y', \quad y(0) = 1, \quad y'(0) = \frac{1}{3}.$$

Find $y(3)$.

A. $y(3) = e^3 + 1$

B. $y(3) = e^3$

C. $y(3) = 2e^2 + 1$

D. $y(3) = e + 1$

E. $y(3) = e$

9. Which of the following are true statements about the matrix $A = \begin{bmatrix} 3 & 9 & 1 \\ 2 & 6 & 7 \\ 1 & 3 & -6 \end{bmatrix}$?

I. $E_0 = \begin{bmatrix} 1 & 3 & -6 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ is a row echelon form of A.

II. $E_0 = \begin{bmatrix} 1 & 3 & -6 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ and $E_1 = \begin{bmatrix} 1 & 3 & -6 \\ 0 & 0 & 19 \\ 0 & 0 & 0 \end{bmatrix}$ are row echelon forms of A.

III. $E_0 = \begin{bmatrix} 1 & 3 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ is the reduced row echelon form of A.

IV. $E_0 = \begin{bmatrix} 1 & 3 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ and $E_1 = \begin{bmatrix} 1 & 3 & -6 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$ are both reduced row echelon forms of A.

A. Only I is true

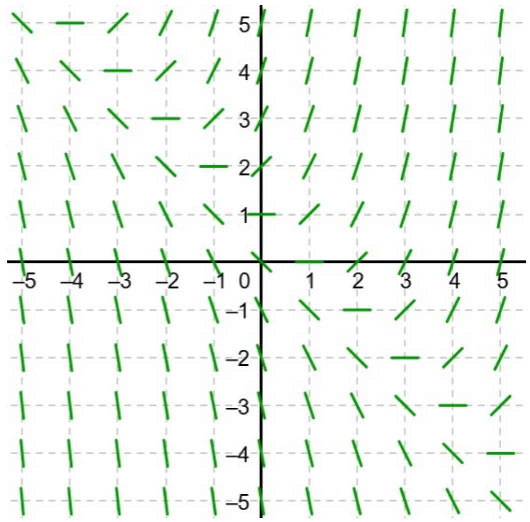
B. Only II is true

C. Only II and III are true

D. Only IV is true

E. Only I, II and III are true.

1. The slope field shown below is the graphical representation of which of the following differential equations?



A. $\dfrac{dy}{dx} = x + y - 1$

B. $\dfrac{dy}{dx} = x - y - 1$

C. $\dfrac{dy}{dx} = y - x - 1$

D. $\dfrac{dy}{dx} = x + y$

E. $\dfrac{dy}{dx} = y - x$

12. Find the solution of the initial value problem

$$y' = -\sin^2(x+y),$$
$$y(0) = \frac{\pi}{4}$$

A. $y(x) = \tan^{-1}(x+1) - x$

B. $y(x) = \tan^{-1}(1-x) + x$

C. $y(x) = \tan^{-1}(2x+1) - 2x$

D. $y(x) = \tan^{-1}(3x+1) - x$