Review Problems for the MA262 Final Exam

9. Let A_{ij} be the cofactor of the element a_{ij} in the matrix $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ with $\det(A) = 5$. The value of the expression $a_{11}A_{11} + a_{12}A_{12} + a_{21}A_{21} + a_{22}A_{22}$ then is equal to

That is: A_{ij} is what we called the cofactor C_{ij}

A. 0

B. 5

C. 10

D. 15

E. undetermined by the information given above

10. Let $A = \begin{bmatrix} 1 & k \\ k & 3 \end{bmatrix}$ and $b = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$. The values of k for which the system $A\mathbf{x} = b$ has a unique solution are

A. $k \neq \sqrt{6}$

B. $k \neq \{\sqrt{3}, -\sqrt{3}\}$

C. $k = \{\sqrt{2}, -\sqrt{2}\}$

D. All real numbers

E. None of the above

2. A ranger in a national park found at noon the body of a wild boar killed by a poacher. To convict a suspected poacher, one needs to know the time when the boar was killed. To determine this time, the temperature of the body was measured twice: at noon it was 60° F and at 1 p. m. it was 55° F. The air temperature on that day was 50° F and it did not change since 8 a.m. The normal body temperature of the boar when it is alive is 90° F. When was the boar killed?

A. 6 a.m.

B. 10 a.m.

C. 10:30 a.m.

D. 11 a.m.

E. 11:30 a.m.

Remember: Newton's Law of Cooling:

$dT/dt = k(A - T)$

— T = temp,
— A = ambient temp,
— k unknown const.

11. Consider the vectors $\mathbf{v}_1 = \begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}$, $\mathbf{v}_2 = \begin{bmatrix} 3 \\ 1 \\ 7 \\ 3 \end{bmatrix}$, $\mathbf{v}_3 = \begin{bmatrix} 5 \\ -3 \\ 9 \\ 1 \end{bmatrix}$, $\mathbf{v}_4 = \begin{bmatrix} -2 \\ 4 \\ 2 \\ 8 \end{bmatrix}$. The dimension of the space span$\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3, \mathbf{v}_4\}$ is then equal to

A. 1

B. 2

C. 3

D. 4

E. 5.

8. Find an implicit solution of the initial value problem:

$$\begin{cases} (6x^2y^2 + 4e^x - 2y\sin 2x) + (4x^3y + \cos 2x)\dfrac{dy}{dx} = 0, \\ y(0) = 1 \end{cases}.$$

A. $2x^3y^2 - y\cos 2x + 4e^x = 3$

B. $x^3y^2 + y\cos 2x + 4ye^x = 0$

C. $2x^3y^2 + y\cos 2x + 4e^x = 5$

D. $x^3y^2 + y^2\cos 2x + 4e^x = 5$

E. $2x^3y^2 + y\cos 2x - 4ye^x = -3$

3. The general solution of $xy' - y = x^2e^x$ is

A. $y = xe^x + cx$

B. $y = x^2e^x - xe^x + cx$

C. $y = xe^x - cx^2$

D. $y = x^2e^x + xe^x + cx$

E. None of the above

6. The solution of

$$y' + \frac{y}{x} = \frac{2}{x^2 y}, \qquad x \neq 0$$

is given by

A. $x^2y^2 + 4xy = C$

B. $x^2y^2 + 4x = C$

C. $xy^2 - 2x = C$

D. $x^2y^2 - 4x = C$

E. $xy^2 - 4x = C$

3. Find the explicit solution of the initial value problem

$$y' = \frac{xy^2}{x^2+1} \quad , \quad y(0) = 3.$$

A. $\frac{1}{2}(6 + \ln(1+x^2))$

B. $\dfrac{6}{2 - 3\ln(1+x^2)}$

C. $\dfrac{6}{2 + 3\ln(1+x^2)}$

D. $\frac{1}{2}(6 - 3\ln(1+x^2))$

E. $\frac{1}{3}(9 - 2\ln(1+x^2))$

8. Which of the following is the general solution to $y'' + 4y = e^{2t} + 12\sin(2t)$?

A. $y(t) = c_1\cos(2t) + c_2\sin(2t) + \dfrac{1}{8}e^{2t} - 3t\cos(2t)$

B. $y(t) = c_1 e^{2t} + c_2 e^{-2t} + \dfrac{1}{4}t\,e^{2t} - 3t\cos(2t)$

C. $y(t) = c_1 + c_2 e^{-4t} + \dfrac{1}{12}te^{2t} - 3t\cos(2t)$

D. $y(t) = c_1\cos(2t) + c_2\sin(2t) + \dfrac{1}{8}e^{2t} + 3\sin(2t)$

E. None of the above.

10. A particular solution of the equation

$$y^{(4)} - y'' = 2\sin t - 3e^{-t} + 4t$$

is of the form

A. $y_p(t) = A\cos t + B\sin t + Cte^{-t} + t^2(Dt + E)$

B. $y_p(t) = t(A\cos t + B\sin t) + Cte^{-t} + t^2(Dt + E)$

C. $y_p(t) = A\cos t + B\sin t + Ce^{-t} + t^2(Dt + E)$

D. $y_p(t) = t(A\cos t + B\sin t) + Ce^{-t} + t^2(Dt + E)$

E. $y_p(t) = t(A\cos t + B\sin t) + Cte^{-t} + t(Dt + E)$

9. According to the method of undetermined coefficients, what is the proper form of a particular solution Y to the following differential equation?

$$y^{(4)} - 4y'' = 24t^2 - 4 - 3te^t.$$

A. $Y(t) = At^2 + Bte^t$.

B. $Y(t) = At^2 + Bt + C + Dte^t + Ee^t$.

C. $Y(t) = At^3 + Bt^2 + Ct + D + Ete^t + Fe^t$.

D. $Y(t) = At^4 + Bt^3 + Ct^2 + Dte^t + Ee^t$.

E. None of the above.

aka: unstable critical points

5. How many asymptotically unstable equilibrium solution(s) does the following differential equation have?

$$y' = (y^2 + 1)(y^2 - 1)(y + 2).$$

A. 0,

B. 1,

C. 2,

D. 3,

E. None of the above.

3. Find the solution $y(x)$ to

$$\frac{dy}{dx} = e^{-\frac{y}{x}} + \frac{y}{x},$$
$$y(e) = 0.$$

A. $y(x) = \ln(\ln(x))$

B. $y(x) = x\ln(\ln(x))$

C. $y(x) = \ln(\ln(x) + 1) - \ln(2)$

D. $y(x) = x\ln(x) - e$

E. $y(x) = x\ln(x) - x$

14. Using the method of Undetermined Coefficients, which of the following is the correct form of a particular solution y_p to the nonhomogeneous differential equation

$$y^{(4)} - y = xe^x + 3\cos x - 4x \ ?$$

A. $y_p = x(A + Bx)e^x + (C\cos x + D\sin x) + Ex$

B. $y_p = (A + Bx)e^x + (C\cos x + D\sin x) + E + Fx$

C. $y_p = (A + Bx)e^x + x(C\cos x + D\sin x) + x(E + Fx)$

D. $y_p = x(A + Bx)e^x + x(C\cos x) - Dx$

E. $y_p = x(A + Bx)e^x + x(C\cos x + D\sin x) + E + Fx$

9. Find the general solutions to $y'' + 6y' + 10y = 0$

A. $c_1e^{-2t}\cos(t) + c_2e^{-2t}\sin(t)$

B. $c_1e^{-3t}\cos(t) + c_2e^{-3t}\sin(t)$

C. $c_1e^{-3t}\cos(2t) + c_2e^{-3t}\sin(2t)$

D. $c_1e^{2t}\cos(t) + c_2e^{2t}\sin(t)$

E. $c_1e^{3t}\cos(t) + c_2e^{3t}\sin(t)$

7. Find the general solution of the system

$$\mathbf{x}' = \begin{pmatrix} -3 & -4 \\ 1 & 1 \end{pmatrix} \mathbf{x}.$$

A. $c_1 \begin{pmatrix} 2 \\ 1 \end{pmatrix} e^t + c_2 \left[\begin{pmatrix} 2 \\ 1 \end{pmatrix} te^t + \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^t \right],$

B. $c_1 \begin{pmatrix} -2 \\ 1 \end{pmatrix} e^{-t} + c_2 \left[\begin{pmatrix} -2 \\ 1 \end{pmatrix} te^{-t} + \begin{pmatrix} -1 \\ 0 \end{pmatrix} e^{-t} \right],$

C. $c_1 \begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{-t} + c_2 \left[\begin{pmatrix} 2 \\ 1 \end{pmatrix} te^{-t} + \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{-t} \right],$

D. $c_1 \begin{pmatrix} -2 \\ 1 \end{pmatrix} e^{-t} + c_2 \left[\begin{pmatrix} -2 \\ 1 \end{pmatrix} te^{-t} + \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{-t} \right],$

E. $c_1 \begin{pmatrix} -2 \\ 1 \end{pmatrix} e^t + c_2 \left[\begin{pmatrix} -2 \\ 1 \end{pmatrix} te^t + \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^t \right].$

2. Find the number of **stable** critical points for the autonomous equation

$$\frac{dx}{dt} = x(x-1)^2(x+3)(x^2-4).$$

A. 1

B. 2

C. 3

D. 4

E. 0

16. One eigenvalue of $\begin{bmatrix} 1 & 2 & 2 \\ -1 & 4 & 1 \\ 0 & 0 & 3 \end{bmatrix}$ is $\lambda = 3$. A basis for the corresponding eigenspace is

A. $\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} \right\}$

B. $\left\{ \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$

C. $\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix} \right\}$

D. $\left\{ \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix} \right\}$

E. $\left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \right\}$

17. If $y(x)$ is a solution of $y'' - 2y' + y = 0$ satisfying $y(0) = 1$ and $y'(0) = -1$, then $y(\frac{1}{2}) =$

A. 0

B. $-e^{\frac{1}{2}}$

C. $e^{\frac{1}{2}}$

D. $2e^{\frac{1}{2}}$

E. $-3e^{\frac{1}{2}}$

24. The general solution of $\mathbf{x}' = \begin{bmatrix} 3 & 5 \\ -1 & -1 \end{bmatrix} \mathbf{x}$ has the form

A. $c_1 e^t \begin{bmatrix} 2\cos t - \sin t \\ -\cos t \end{bmatrix} + c_2 e^t \begin{bmatrix} \cos t + 2\sin t \\ -\sin t \end{bmatrix}$

B. $c_1 e^t \begin{bmatrix} \cos t \\ 2\cos t - \sin t \end{bmatrix} + c_2 e^t \begin{bmatrix} -\sin t \\ \cos t - 2\sin t \end{bmatrix}$

C. $c_1 e^t \begin{bmatrix} 2\cos t + \sin t \\ -\cos t \end{bmatrix} + c_2 e^t \begin{bmatrix} \cos t + 2\sin t \\ \sin t \end{bmatrix}$

D. $c_1 e^t \begin{bmatrix} \cos t - 2\sin t \\ \sin t \end{bmatrix} + c_2 e^t \begin{bmatrix} 2\cos t + \sin t \\ -\cos t \end{bmatrix}$

E. None of the above.

Use method of variation of parameters from [L29]

7. If $y = u_1y_1 + u_2y_2$ where $y_1 = e^{2x}$ and $y_2 = e^{-2x}$ is a particular solution of

$$y'' - 4y = 4\tan x$$

then u_1 and u_2 are determined by

A. $u_1' = e^{-2x}\tan x \quad u_2' = -e^{2x}\tan x$

B. $u' = -e^{-2x}\sec^2 x \quad u_2' = e^{2x}\sec^2 x$

C. $u_1' = -2\sin 2x\tan x \quad u_2' = 2\cos 2x\tan x$

D. $u_1' = 2\sin 2x\tan x \quad u_2' = -2\cos 2x\tan x$

E. $u_1' = \tan x \quad u_2' = 0$

10. Find all values of a such that the following system of equations has exactly one solution.

$$\begin{cases} x + y - z = 2 \\ x + 2y + z = 3 \\ x + y + (a^2 - 5)z = a \end{cases}$$

A. $a \neq 2$ and $a \neq -2$

B. $a = 2$ or $a = -2$

C. $a = -2$

D. $a = \pm\sqrt{5}$

E. $a \neq 0$

17. Determine all values of k such that the vectors $(1, -1, 0)$, $(1, 2, 2)$, $(0, 3, k)$ are a basis for $\mathbb{R}^3$.

A. $k = 1$

B. $k = 2$

C. $k \neq 2$

D. $k \neq 1$

E. $k \neq 3$

12. Let $y(x)$ satisfy

$$y'' - y' = 2\sin(x),$$
$$y(0) = 3, \quad y'(0) = 0.$$

Then $y(\pi)$ is equal to:

A. $e^{\pi} + 10$

B. $e^{\pi} + 1$

C. $e^{\pi} + 2$

D. $e^{\pi} + 3$

E. e^{π}

12. For the system $\mathbf{x}' = \begin{bmatrix} 5 & 5 \\ -8 & -7 \end{bmatrix} \mathbf{x}$, the origin is

A. a saddle point

B. a proper node source

C. a center point

D. a spiral sink

E. a spiral source

15. The solution curves of the system

$$X'(t) = \begin{bmatrix} 1 & 1 \\ 1 & -2 \end{bmatrix} X(t),$$

where $X(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$, have the following structure near the origin $x_1 = x_2 = 0$:

A. A saddle point

B. A nodal sink

C. A nodal source

D. A center

E. parallel lines.

16. Let $X(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$. Find all the values of a so that the origin $x_1 = x_2 = 0$

$$X'(t) = \begin{bmatrix} a & -27 \\ 3 & 0 \end{bmatrix} X(t)$$

is a spiral source of the system, that is, the solution curves are spirals approaching infinity as $t \to \infty$?

A. $0 < a < 27$

B. $0 < a < 18$

C. $-18 < a < 18$

D. $0 > a > -18$.

E. $0 > a > -27$.

17. Let $X(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$ satisfy

[Compute the particular solution X(t) = (x1(t), x2(t)), and then compute that limit of x2'/x1']

$$X'(t) = \begin{bmatrix} 1 & 6 \\ 1 & 2 \end{bmatrix} X(t), \quad X(0) = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$$

The limit $S = \lim_{t\to\infty} \dfrac{x_2'(t)}{x_1'(t)}$, which is the slope of the curve $X(t)$ as $t \to \infty$, is equal to

A. $S = -3$

B. $S = \dfrac{1}{2}$

C. $S = -\dfrac{1}{3}$

D. $S = -\dfrac{1}{2}$

E. $S = 3$

aka: the eigenvalue(s) have defect d=0

15. Which of the following matrices are nondefective:

$$A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}, \quad C = \begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}?$$

A. A, B

B. A, C

C. B only

D. C only

E. A only

22. The eigenvalues of the matrix $A = \begin{bmatrix} 3 & 0 & 0 \\ -2 & 3 & -2 \\ 2 & 0 & 5 \end{bmatrix}$ are 3 and 5. One of the associated eigenspaces has dimension one. This eigenspace has basis:

A. $\begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$

B. $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$

C. $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

D. $\begin{bmatrix} 1 \\ 0 \\ -1 \end{bmatrix}$

E. $\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$

8. Let $A = \begin{bmatrix} -1 & 0 & 0 \\ 1 & 5 & -1 \\ 1 & 6 & -2 \end{bmatrix}$. Let λ_1, λ_2 and λ_3 denote the eigenvalues of A and let E_1, E_2 and E_3 denote the corresponding eigenspaces. Which of the following is correct?

A. $\lambda_1 = 2$, $\lambda_2 = 3$ and $\lambda_3 = -1$, $\dim E_1 = \dim E_2 = \dim E_3 = 1$

B. $\lambda_1 = \lambda_2 = -1$, $\lambda_3 = 4$, $\dim E_1 = \dim E_2 = 2$, $\dim E_3 = 1$

C. $\lambda_1 = \lambda_2 = -1$, $\lambda_3 = 4$, $\dim E_1 = \dim E_2 = 1$, $\dim E_3 = 1$

D. $\lambda_1 = 1$, $\lambda_2 = \lambda_3 = 4$, $\dim E_1 = 1$, $\dim E_2 = \dim E_3 = 2$

E. $\lambda_1 = \lambda_2 = -1$, $\lambda_3 = 3$, $\dim E_1 = \dim E_2 = 2$, $\dim E_3 = 1$

15. Let A be a 2×2 matrix whose entries are real numbers. If $\lambda = 2 + 3i$ is a complex eigenvalue of A with corresponding complex eigenvector $\mathbf{w} = \begin{bmatrix} 1-i \\ 4 \end{bmatrix}$, then the general solution to $\mathbf{x}' = A\,\mathbf{x}$ is:

A. $\mathbf{x} = C_1\, e^{2t} \begin{bmatrix} \cos 3t + \sin 3t \\ 4\cos 3t \end{bmatrix} + C_2\, e^{2t} \begin{bmatrix} \sin 3t - \cos 3t \\ 4\sin 3t \end{bmatrix}$

B. $\mathbf{x} = C_1\, e^{2t} \begin{bmatrix} \cos 3t - \sin 3t \\ 4\cos 3t \end{bmatrix} + C_2\, e^{2t} \begin{bmatrix} \sin 3t - \cos 3t \\ 4\sin 3t \end{bmatrix}$

C. $\mathbf{x} = C_1\, e^{3t} \begin{bmatrix} \cos 2t + \sin 2t \\ 4\cos 2t \end{bmatrix} + C_2\, e^{3t} \begin{bmatrix} \sin 2t - \cos 2t \\ 4\sin 2t \end{bmatrix}$

D. $\mathbf{x} = C_1\, e^{2t} \begin{bmatrix} \cos 3t + \sin 3t \\ 4\cos 3t \end{bmatrix} + C_2\, e^{2t} \begin{bmatrix} \sin 3t + \cos 3t \\ 4\sin 3t \end{bmatrix}$

E. $\mathbf{x} = C_1\, e^{2t} \begin{bmatrix} \cos 3t + \sin 3t \\ 4\cos 3t \end{bmatrix} + C_2\, e^{2t} \begin{bmatrix} \sin 3t - \cos 3t \\ -4\sin 3t \end{bmatrix}$

17. Let $\mathbf{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$ be the solution of the system of differential equations

$$\mathbf{x}' = A\mathbf{x}, \quad A = \begin{bmatrix} 4 & -3 \\ 6 & -7 \end{bmatrix}.$$

with the initial data $\mathbf{x}(0) = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$. Given that the eigenvalues of A and their corresponding eigenvectors of A are $\lambda_1 = -5$, $V_1 = \begin{bmatrix} 1 \\ 3 \end{bmatrix}$ and $\lambda_2 = 2$, $V_2 = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$, find $x_1(1)$.

A. $3e^2$

B. $-2e^{-5}$

C. e^{-5}

D. $-3e^{-5}$

E. $e^{-5} + 3e^2$

19. Find all constants b such that the origin is a spiral source of the system

$$X'(t) = \begin{bmatrix} 3 & b \\ 1 & 4 \end{bmatrix} X(t), \quad b \text{ in } \mathbb{R}$$

are

A. $b < -\frac{1}{3}$

B. $b > -\frac{1}{3}$

C. $-\frac{1}{4} < b < -\frac{1}{4}$

D. $b > -\frac{1}{4}$

E. $b < -\frac{1}{4}$

16. Given that $\lambda = 1$ is a defective eigenvalue of the matrix $\begin{bmatrix} 2 & -1 \\ 1 & 0 \end{bmatrix}$, which of the following is the solution of the initial value problem:

$$\mathbf{x}' = \begin{bmatrix} 2 & -1 \\ 1 & 0 \end{bmatrix} \mathbf{x} \quad \mathbf{x}(0) = \begin{bmatrix} 4 \\ 2 \end{bmatrix}?$$

A. $\mathbf{x}(t) = 2e^t \begin{bmatrix} 1 \\ 1 \end{bmatrix} + e^t \left\{ t \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 2 \\ 0 \end{bmatrix} \right\}$

B. $\mathbf{x}(t) = 2e^t \begin{bmatrix} 1 \\ 1 \end{bmatrix} + 2te^t \begin{bmatrix} 1 \\ 0 \end{bmatrix}$

C. $\mathbf{x}(t) = 2e^t \begin{bmatrix} 1 \\ -1 \end{bmatrix} + 2e^t \left\{ t \begin{bmatrix} 1 \\ -1 \end{bmatrix} + \begin{bmatrix} 1 \\ 2 \end{bmatrix} \right\}$

D. $\mathbf{x}(t) = 2e^t \begin{bmatrix} 1 \\ 1 \end{bmatrix} + 2e^t \left\{ t \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right\}$

E. $\mathbf{x}(t) = e^t \begin{bmatrix} 1 \\ 1 \end{bmatrix} + e^t \left\{ t \begin{bmatrix} 1 \\ 1 \end{bmatrix} + \begin{bmatrix} 3 \\ 1 \end{bmatrix} \right\}$

20. The general solution of the system

$$\mathbf{x}' = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix} \mathbf{x}$$

is given by

A. $\mathbf{x}(t) = ae^{t}\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + be^{t}\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + ce^{2t}\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$

B. $\mathbf{x}(t) = ae^{t}\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + be^{-t}\begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix} + ce^{2t}\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$

C. $\mathbf{x}(t) = ae^{t}\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} + be^{t}\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + ce^{2t}\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}$

D. $\mathbf{x}(t) = ae^{t}\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + be^{t}\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} + ce^{2t}\begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}$

E. $\mathbf{x}(t) = ae^{2t}\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + be^{2t}\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + ce^{t}\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$