Which of these statements is true for <u>all</u> 5×5 matrices $\underline{A}, \underline{B}$?

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1) $\det(\underline{A} + \underline{B}) = \det(\underline{A}) + \det(\underline{B})$?

$$\det\left(\begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}\right) = \det\left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right) + \det\left(\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}\right)$$

$$0 \qquad\qquad 1 \quad + \quad -1$$

$$\det\left(\begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}\right) \stackrel{?}{=} \det\left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right) + \det\left(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}\right)$$

$$4 \quad \neq \quad 1 \quad + \quad 1$$

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2) $\det(\underline{A}^T) = -\det(\underline{A})$

False: $\det(\underline{A}) = \det(\underline{A}^T)$ always (as long as they are square)

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3) $\underline{A}\underline{B} = \underline{\emptyset}$ implies that $\underline{A} = \underline{\emptyset}$ or $\underline{B} = \underline{\emptyset}$?

<u>False</u>: $\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

4) if ~~two~~ $\det(\underline{A}) = 0$ then two rows must be proportional ?

False: example in 5×5: $\det(\underline{A}) = 0$ means the rows of $\underline{A}$ are linearly dependent

If there are 5 rows, then they can be dependent if adding 3 or more adds to 0, but they don't have to be multiples of each other.

E) If $\underline{A}$ is 5×5, then is

$$\det(-\underline{A}) = -\det(\underline{A})\,?$$

$-\underline{A}$ multiplies all 5 rows of $\underline{A}$ by (-1), making

$$\det(-\underline{A}) = (-1)^5 \det(\underline{A}) = -\det(\underline{A}).$$

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Assume $\underline{A}$ is 4×4 and is row equiv. to

$$\underline{B} = \begin{bmatrix} 1 & 0 & 2 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

which is true?

X 1) $\underline{B}$ is not the red. echelon form of $\underline{A}$.

X 2) $\det(\underline{A}) \neq 0$.

~~rows~~ ops	effect on determinant
• swap row/col	$\times\ (-1)$
• multiply row or column by k	$\times\ k$
• $R_i \to R_i + kR_j$, $C_i \to C_i + kC_j$	$\times\ 1$ (no effect)

$$\det(\underline{A}) = D \longrightarrow \underbrace{D\,\boxed{(-1)\cdot(+1)\cdot(3)\cdot(-7)\cdot(-1)}}_{\det(\underline{B})}$$

Thm If $\underline{A}$ is row equiv. to $\underline{B}$, then

$$\det(\underline{A}) = k \det(\underline{B}) \text{ for some } k$$

Here $\det(\underline{B}) = 0$. $\longrightarrow$ $\det(\underline{A}) = (\#)\det(\underline{B})$

$$= (\#)\cdot 0 = 0$$

$$\det(\underline{A}) = 0.$$

$$\underline{B} = \begin{bmatrix} \boxed{1} & 0 & 2 & 0 \\ 0 & \boxed{1} & 1 & 0 \\ 0 & 0 & 0 & \boxed{1} \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

c) The null space of $\underline{A}$ has dimension 3

To find null$(\underline{A})$, solve $\underline{Ax} = \underline{\emptyset}$

row reduce $[\underline{A} \,\vdots\, \underbrace{\underline{\emptyset}}_{4\times 1 \text{ vector.}}]$

$$\ldots \quad [\text{RREF } A \,\vdots\, \underline{\emptyset}] = \left[\begin{array}{cccc|c} \boxed{1} & 0 & 2 & 0 & 0 \\ 0 & \boxed{1} & 1 & 0 & 0 \\ 0 & 0 & 0 & \boxed{1} & 0 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

$\uparrow$ free

$$x_1 + 2x_3 = 0$$
$$x_2 + x_3 = 0$$
$$x_4 = 0$$
$$x_3 \text{ free} = s.$$

$$x_1 = -2s$$
$$x_2 = -s$$
$$x_4 = 0$$
$$x_3 = s$$

$$\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} = \begin{bmatrix} -2s \\ -s \\ s \\ 0 \end{bmatrix} = s \begin{bmatrix} -2 \\ -1 \\ 1 \\ 0 \end{bmatrix}$$

$$\text{null}(\underline{A}) = \text{span}\left\{ \begin{bmatrix} -2 \\ -1 \\ 1 \\ 0 \end{bmatrix} \right\}$$

$$\dim(\text{null}(\underline{A})) = 1.$$

$$(\#\text{ cols}) = (\#\text{ vars}) = \underbrace{(\#\text{ pivots vars})}_{\text{rank} = \#\text{pivots}} + \underbrace{(\#\text{ free vars})}_{\text{dim of null}(\underline{A})}$$

$$\underline{B} = \begin{bmatrix} \boxed{1} & 0 & 2 & 0 \\ 0 & \boxed{1} & 1 & 0 \\ 0 & 0 & 0 & \boxed{1} \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

(D) col($\underline{A}$) has dimension 2.

False, rank($\underline{A}$) = 3.

(E) If $\underline{A}x = \underline{b}$ ~~ha~~ is consistent for some $\underline{b} \neq 0$ then $\underline{A}x = \underline{b}$ has infinitely many sols.

True: Solving $\underline{A}x = \underline{b}$

row reducing $\left[\begin{array}{c|c} \underline{A} & \begin{matrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{matrix} \end{array}\right]$

... $\left[\begin{array}{cccc|c} 1 & 0 & 2 & 0 & \tilde{b}_1 \\ 0 & 1 & 1 & 0 & \tilde{b}_2 \\ 0 & 0 & 0 & 1 & \tilde{b}_3 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right] \leftarrow 0 = \tilde{b}_4$

$x_1 + 2x_3 = \tilde{b}_1$

$x_2 + x_3 = \tilde{b}_2$

$\boxed{x_3 \text{ free}}$

$x_4 = \tilde{b}_3$

$$\underline{A} = \begin{bmatrix} 2 & -1 & -2 \\ -4 & 2 & 4 \\ -8 & 4 & 8 \end{bmatrix}$$

basis of null($\underline{A}$).

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for all 4×2, what must be true?

X A) dim(row($\underline{A}$)) > dim(col($\underline{A}$))?

B) columns linearly dep.?

no ex: $\begin{bmatrix} \boxed{1} & 0 \\ 0 & \boxed{1} \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$

[1 0]
[0 1]
[0 0]
[0 0]

C) rows must be linearly depen.?

rows are like $\mathbb{R}^2$ vectors.

4 of them, $4 > 2 \Rightarrow$ lin depen.

$$\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \right\}$$

$\underline{u}$ $\quad$ $\underline{v}$

$k\underline{u} = \underline{v}$

$a\underline{u} + b\underline{v} = \emptyset$ $\quad$ $(0, 0)$

$0\underline{u} + 1\underline{v} = \emptyset$

☆ all vectors are dependent with $\emptyset$ vector.

(D) (4×2) columns must be lin indep

$$\begin{bmatrix} \vdots & \vdots \end{bmatrix}$$

False $\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$ $\begin{bmatrix} 1 & 1 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$

(E) rows must be indep

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$$