Outline for "vector spaces" and "subspaces"

Before introducing vocab, I want to roughly discuss the setup and goals of "vector spaces" and "subspaces."

Setup: Have "sets" of things to describe, like:

- sets of vectors / matrices
- sets of points in $\mathbb{R}^2$, $\mathbb{R}^3$, $\mathbb{R}^4$, ... (lines, planes, ...)
- sets of solution functions for ODEs.

("all solutions of $y''+y=0$" $= \{ y(x) : y''+y=0 \}$)

Goal: Find "minimal generating sets" for these sets

- $\{\hat{i}, \hat{j}\}$ is a "min. gen. set" for $\mathbb{R}^2$
- $\{\hat{i}, \hat{j}, \hat{k}\}$ " " " " " " $\mathbb{R}^3$
- $\{\cos(x), \sin(x)\}$ is a "min.gen. set" for set of sol. functions of ODE $y''+y=0$.

- Need to define/clarify "generating set" and "minimal"
- ★ Need to ensure these "minimal gen. sets" even exist!

The idea of a "minimal generating set" is intuitive enough ATM that we needn't worry about a precise definition yet.

So, Q: Which sets have "minimal generating sets"?

A: Sets which are "Vector Spaces" and/or "Subspaces"