Sec 2.2 Stability and equilibrium

ODE is autonomous if $\frac{dy}{dt} = f(y)$ ← no explicit "t"/depen. var.

Population models $\frac{dP}{dt} = kP$, $\frac{dP}{dt} = kP(M-P)$ are autonomous eqs.

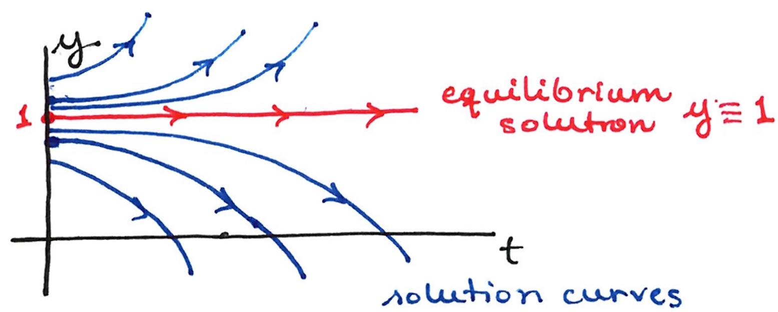
Def: Critical point of an autonomous eq are values $y=c$ (or $P=c, \ldots$) such that $f(y)=0$ ($f(c)=0$, so $\frac{dy}{dt}=0$)

Ex $\frac{dy}{dt} = y-1$

$f(y) = y-1$;

critical point @ $y=1$.

$$\begin{cases} y<1 \Longrightarrow y'<0 \\ y=1 \Longrightarrow y'=0 \\ y>1 \Longrightarrow y'>0 \end{cases}$$

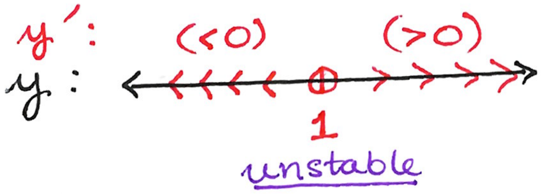


(separable, solutions are $y(t) = 1 + (y_0 - 1)e^t$, where $y(0)=y_0$)

★ If $y=c$ is a critical point of an autonomous eq, then $y(t) \equiv c$ is an equilibrium solution.

Def iff solution curves are "repelled away" from crit. pt. $y=c$ on both sides, then crit. point. $y=c$ is an unstable critical point.

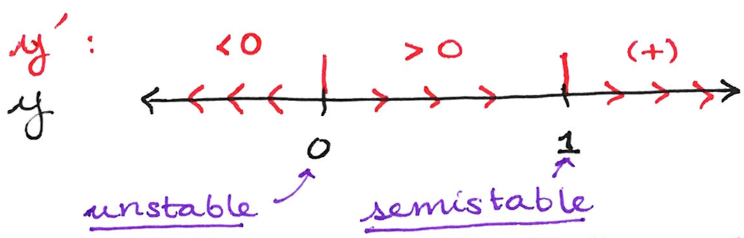
The above example/scenario can be visually described using a <u>phase diagram</u> : $y' = y - 1$



<u>Ex:</u> $\{ y' = y(y-1)^2 \}$

Critical points @ $y = 0$, $y = 1$

so equilibrium solutions $y(t) \equiv 0$, $y(t) \equiv 1$



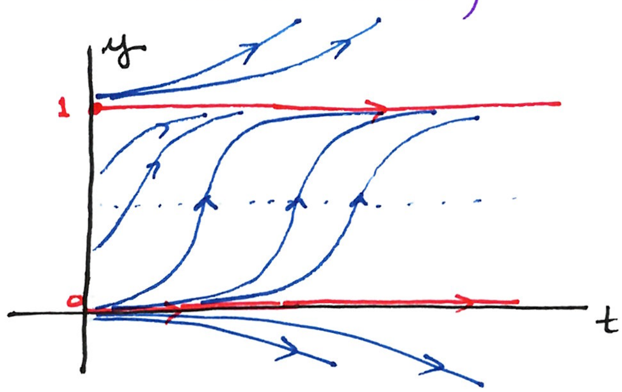
test points

$y = -1 \Rightarrow y' = (-)(-)^2 = (-)$

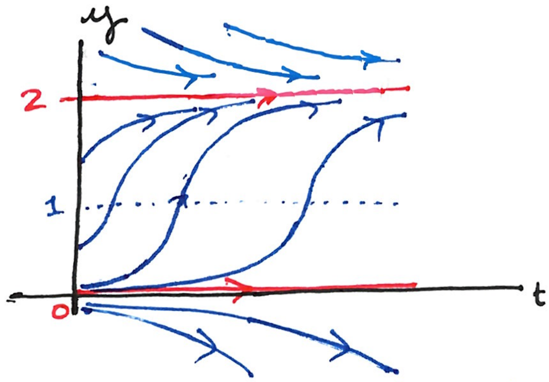
$y = \frac{1}{2} \Rightarrow y' = (+)$

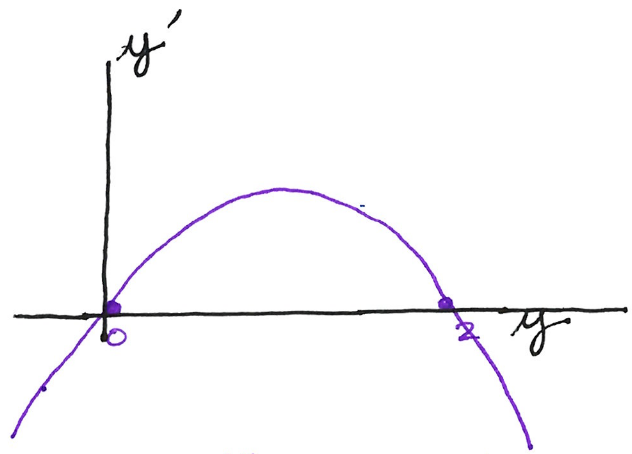
$y = 2 \Rightarrow y' = (+)$

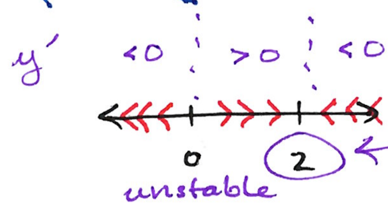
(b/c we have "away" on one side, and "toward" on the other)



"quick example" $\quad y' = 3y(2-y) = 6y - 3y^2$







"toward" on both sides means $y = 2$ is a stable critical point.

Ex $\quad \dfrac{dy}{dt} = (y-1)(y+2)(y^2-9)$

critical points are $y = 1, -2, -3, 3$