(L11) Numerical Methods (Euler's Method)

Given ODE $\left\{ \frac{dy}{dx} = f(x,y) \right\}$, a solution $y = g(x)$

determines/is same as a solution curve $(x, g(x))$

with points $(x_0, y(x_0))$, $(x_1, y(x_1))$, ...

- Unfortunately, finding "true" solutions for ODE's (thereby finding outputs $y(x)$) is often lengthy / difficult / impossible
- In these cases, we use numerical methods to approx solutions / outputs.

  Ex: Euler's method ("tangent line method")

Euler's Method:

Given $\left\{ \begin{array}{l} \cdot \text{ ODE } \frac{dy}{dx} = f(x,y) \\ \cdot \text{ a "known/true" data point } (x_0, y_0) \\ \cdot \text{ target input } x \text{, that we want to know the output } y(x) \text{, (} y \text{ is the particular solution)} \end{array} \right.$ IVP

Goal $\{$ approximate "true" value $y(x)$ $\}$

Specify $\{$ Step size h $\}$

Algorithm $\{$. Set $x_1 = x_0 + h$

$y_1 = y_0 + f(x_0, y_0) h$

If $x_1 = x$ (target) then STOP, and $y_1 \approx y(x)$ we hope.

Else

Set $x_2 = x_1 + h$

$y_2 = y_1 + f(x_1, y_1)h$ $\longrightarrow$ makes point (x_2, y_2)

Continue with x_n = target x;

then $x_n = x_{n-1} + h = x$

$y_n = y_{n-1} + f(x_{n-1}, y_{n-1})h$

and y_n should approximate true value $\underbrace{y(x_n)}_{}$ $= y(x)$

For Monday 2/9,

- Read Notes [L12] Review of Linear Systems;
- Read one/two examples of 3-variable systems in Book section 3.1.