Sec 4.1 and 4.2

Def: (Independence) Given $\underline{u}, \underline{v}, \underline{w}$, try to make linear combination $a\underline{u} + b\underline{v} + c\underline{w} = \underline{0}$ (that is, find (a,b,c) making that work).

always have "<u>trivial solution</u>" $(a,b,c) = (0,0,0)$.

1) If that trivial solution is the <u>only</u> way to make $a\underline{u} + b\underline{v} + c\underline{w} = \underline{0}$, then $\underline{u}, \underline{v}, \underline{w}$ are <u>linearly</u> <u>indep.</u>
2) If there is some $(a,b,c) \neq (0,0,0)$ (some 0's are allowed $(1,0,2)$) solution, then $\underline{u}, \underline{v}, \underline{w}$ are linearly dependent.

Note finding (a,b,c) to make $a\underline{u} + b\underline{v} + c\underline{w} = \underline{0}$ is the same as solving system of eqs / vector eq

$$\underbrace{[\underline{u}\ \underline{v}\ \underline{w}]}_{\underline{A}} \underbrace{\begin{bmatrix} a \\ b \\ c \end{bmatrix}}_{\underline{x}} = \underbrace{\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}}_{\underline{0}}$$

Def: a system of eqs / vector eq $\underline{A}\underline{x} = \underline{b}$ is "<u>homogeneous</u>" if $\underline{b} = \underline{0}$.

$\underline{A}\underline{x} = \underline{0}$, change $\underline{x} \to c\underline{x}$

$$\underline{A}(c\underline{x}) = \underline{0} \longrightarrow c(\underline{A}\underline{x}) = \underline{0} \longrightarrow \underline{A}\underline{x} = \underline{0}$$

Notice homogeneous equations $\underline{A}\underline{x} = \underline{0}$ always have (at least) the <u>trivial solution</u> $\underline{x} = \underline{0}$

If system $a\underline{u} + b\underline{v} + c\underline{w} = \underline{0}$ has nontrivial solution (ex: $(\overset{-1}{\cancel{4}}, -1, 2)$)

then $-\underline{u} - \underline{v} + 2\underline{w} = \underline{0} \longrightarrow \underline{w} = \frac{1}{2}\underline{u} + \frac{1}{2}\underline{v}$.

<u>Thm</u>: If $\underline{u}, \underline{v}, \underline{w}$ are 3×1 vectors, we can make square matrix ~~Ax~~ $\underline{A} = [\underline{u}, \underline{v}, \underline{w}]_{(3\times3)}$. If $|\underline{A}| \neq 0$, then vectors are linearly independent. If $|\underline{A}| = 0$, then <u>lin.</u> <u>depen.</u>

Homogeneous equations $\underline{A}\underline{x} = \underline{0}$ always have (at least) the trivial solution $\underline{x} = \underline{0}$.

$$k\,\underline{A}\underline{x} = k\underline{0} -$$
$$k\,\underline{A}\underline{x} = \underline{0}$$
$$\underline{A}\underline{x} = \tfrac{1}{k}\underline{0} = \underline{0}$$

<u>Subspaces of $\mathbb{R}^n$</u> vectors $\underline{x} = \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix}$

Let W be a <u>subset</u> of $\mathbb{R}^n$ (so W is a collection ~~of~~ vectors)

<u>Def</u> W is a <u>subspace</u> of $\mathbb{R}^n$ if 3 things are true:

($\underline{0}$) If zero vector $\underline{0}$ is in W

(CA) ("closed under addition") If $\underline{u}$ and $\underline{v}$ are elements of W then the linear combination $a\underline{u}+b\underline{v}$ is also contained in W. ($\underline{u}, \underline{v} \in W \Rightarrow a\underline{u}+b\underline{v} \in W$ too)

(CS) ("closed under scaling") If $\underline{u}$ is any vector in W, then every scalar multiple $k\underline{u}$ should also be in W.