A tidbit on "implicit solutions"

In class we found that a general, implicit solution for $\frac{dy}{dx} = \frac{4-2x}{3y^2-5}$ is

$$\underbrace{y^3 - 5y + x^2 - 4x}_{F(x,y)} = C$$

Can we somehow double check this is a solution?

Yes, using <u>implicit differentiation:</u>

$$y^3 - 5y + x^2 - 4x = C$$

(impl. diff:) $\quad 3y^2 y' - 5y' + 2x - 4 = 0$

Now solve for y': $\quad y'(3y^2-5) + 2x - 4 = 0$

$$\Rightarrow \quad y'(3y^2-5) = 4-2x$$

$$\Rightarrow \quad y' = \frac{4-2x}{3y^2-5},$$

so this checks out.

For the circle $x^2+y^2=r^2$, can we find <u>what</u> <u>ODE</u> this curve is an implicit solution for?

Sure, implicit differentiation:

$$x^2 + y^2 = r^2$$

$$\Rightarrow \quad 2x + 2yy' = 0$$

$$\Rightarrow \quad y' = \frac{-2x}{2y} = -\frac{x}{y}, \text{ so ODE is } \frac{dy}{dx} = -\frac{x}{y}.$$

Thus, implicit differentiation of the <u>impl. sol.</u> and solving for y' returns the ODE